Precision flavour physics

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TRR 257 - Particle Physics Phenomenology after the Higgs Discovery

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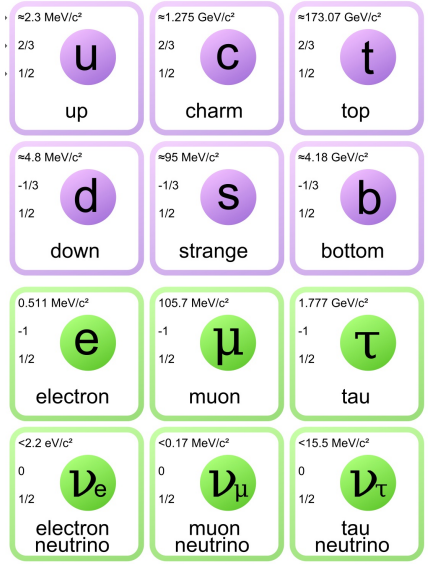
Outline

- Introduction / motivation
- Charged-current semileptonic decays
- Radiative decays
- FCNC semileptonic decays
- Mixing and lifetimes
- Nonleptonic decays
- SMEFT
- Conclusion and outlook

Introduction

- The majority of the SM parameters resides in the Yukawa sector
  - Quarks and leptons are the principal actors of flavour physics
- Many aspects of flavour physics
  - Heavy (top, bottom, charm) and light quarks
  - Mesons and baryons
  - Charged leptons, neutrinos



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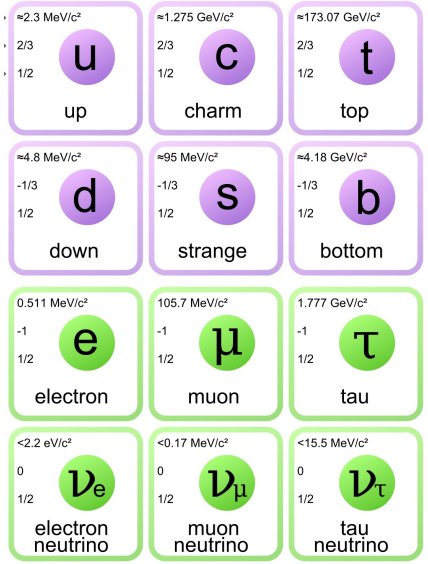
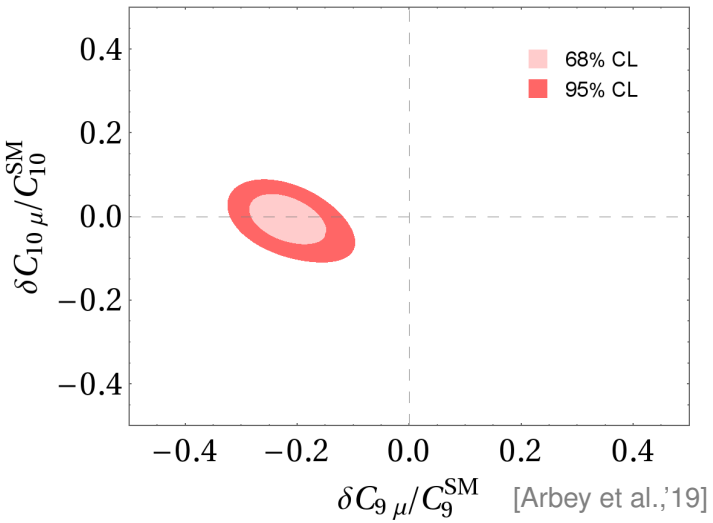


- Have to make a selection:
  Will focus on B mesons, because ...

- Description of B-meson system contains many essential features of (precision) flavour physics
- Global fits driven by B meson anomalies



[Arbey et al.,'19]

Motivation to study flavour

- CP violation
  - Needed to explain matter-antimatter asymmetry (Sakharov conditions)
  - CKM phase is the only established source of CP violation in the SM
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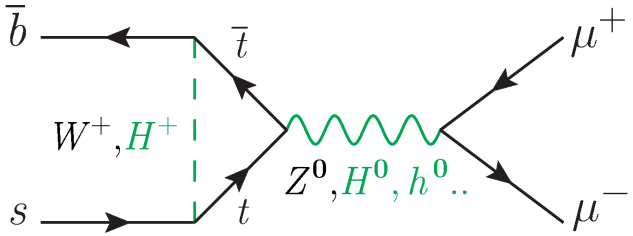
- Indirect search for new physics
  - Look for virtual effects of new phenomena
  - Mostly looked for in rare processes
  - Requires precision in theory and experiment
  - Synergy and complementarity to direct searches



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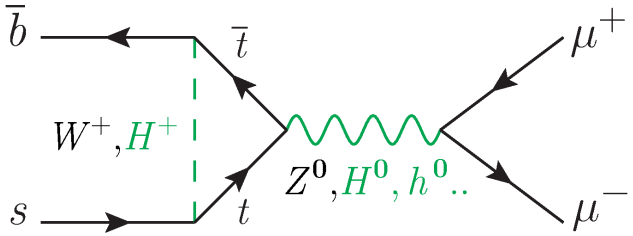
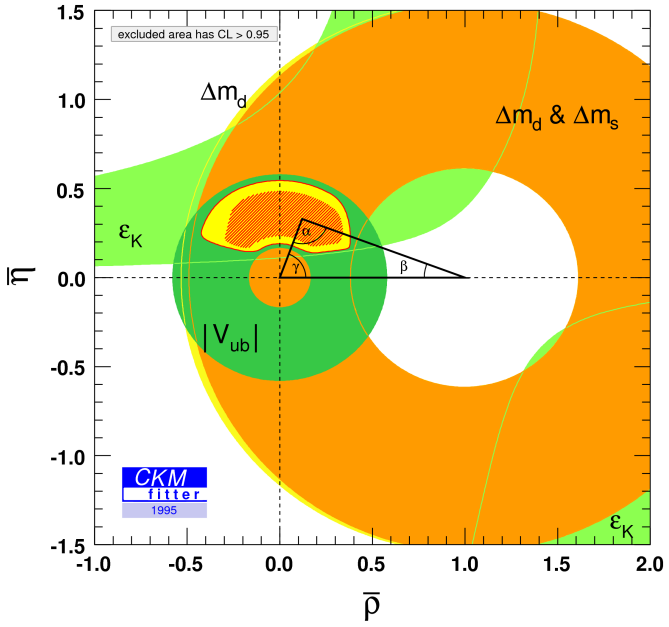


- Huge experimental progress (B-factories, Tevatron, LHC, Belle II, . . .)

Motivation to study flavour

- SM parameters
  - Precise knowledge of masses and mixing parameters needed in all branches of particle physics



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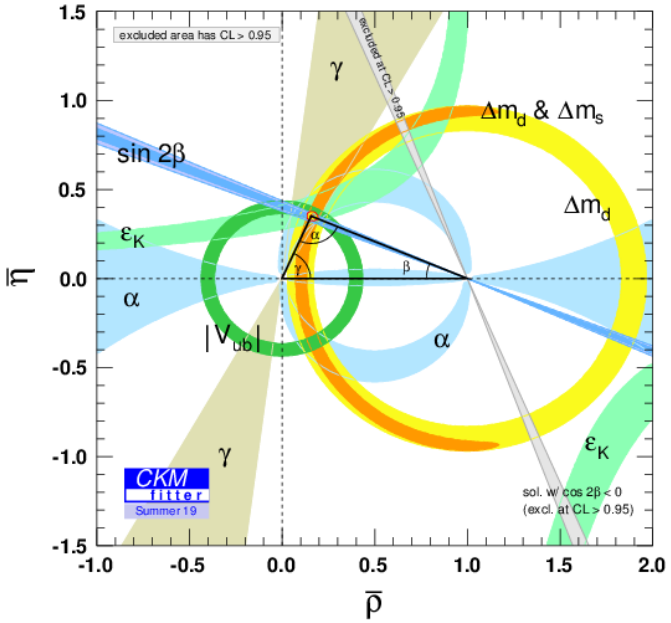
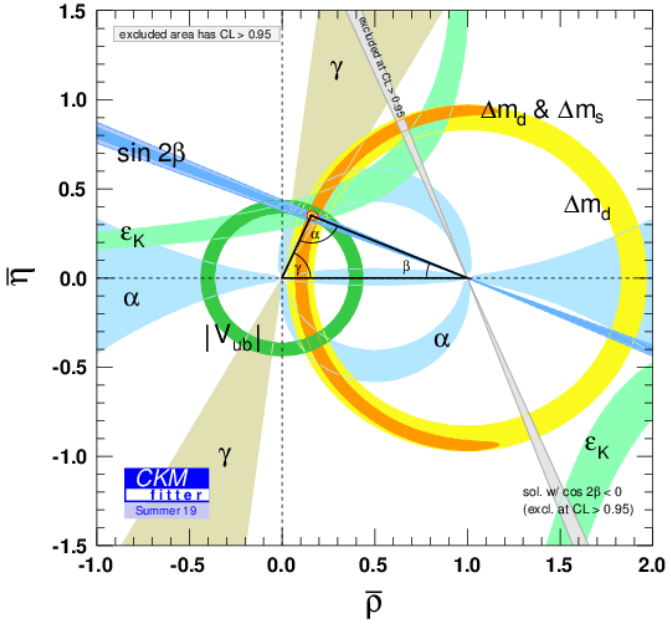
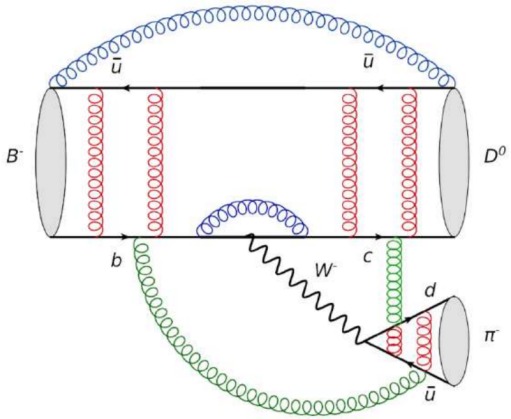


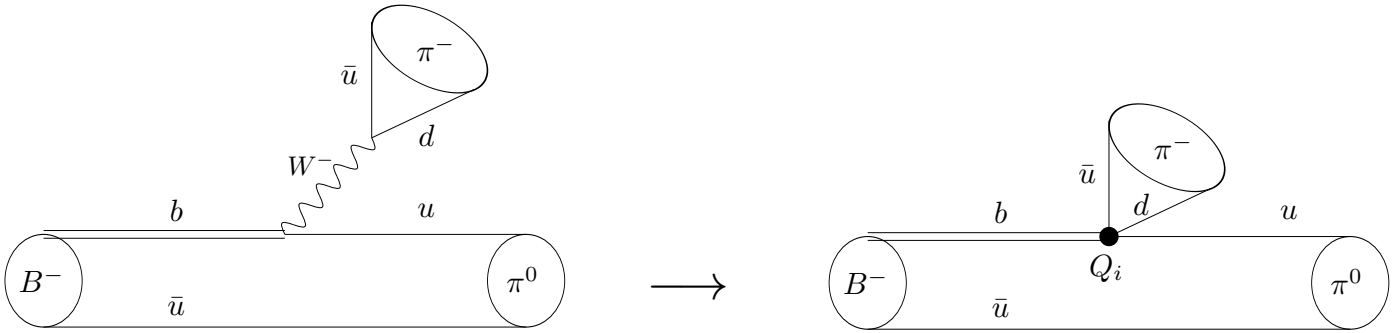
Motivation to study flavour

- SM parameters
  - Precise knowledge of masses and mixing parameters needed in all branches of particle physics
- Problem: confinement of quarks into hadrons
  - Computation of hadronic matrix elements highly non-trivial
  - QCD effects could overshadow the interesting fundamental dynamics
- Need to get control over QCD effects
  - Sophisticated tools available
    - Effective field theories (HQET, SCET,. . .)
    - Heavy-Quark expansion
    - Factorization
    - Perturbative calculations: Loops, . . .
    - Non-pert. techniques: Lattice, Sum rules, . . .
    - Applications also in Higgs, Collider, DM, . . .





Effective theory for B decays



- M_W, M_Z, m_t, $m_H \gg m_b$: integrate out heavy gauge bosons, t-quark, Higgs

- Effective Weak Hamiltonian: [Buras,Buchalla,Lautenbacher'96; Chetyrkin,Misiak,Münz'98]

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{k=3}^{10} C_k Q_k \right] + \text{h.c.}$$

$$Q_1^p = (\bar{d}_L \gamma^\mu T^a p_L)(\bar{p}_L \gamma_\mu T^a b_L) \qquad Q_4 = (\bar{d}_L \gamma^\mu T^a b_L) \textstyle\sum_q (\bar{q} \gamma_\mu T^a q)$$

$$Q_2^p = (\bar{d}_L \gamma^\mu p_L)(\bar{p}_L \gamma_\mu b_L) \qquad Q_5 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q)$$

$$Q_3 = (\bar{d}_L \gamma^\mu b_L) \textstyle\sum_q (\bar{q} \gamma_\mu q) \qquad Q_6 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho T^a b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^a q)$$

$$Q_7 = \frac{e}{16\pi^2} m_b \, \bar{s}_L \, \sigma_{\mu\nu} F^{\mu\nu} b_R \qquad Q_8 = \frac{g_s}{16\pi^2} m_b \, \bar{s}_L \, \sigma_{\mu\nu} G^{\mu\nu} b_R$$

$$Q_9 = (\bar{s}_L \gamma^\mu b_L)(\bar{\ell} \gamma_\mu \ell) \qquad Q_{10} = (\bar{s}_L \gamma^\mu b_L)(\bar{\ell} \gamma_\mu \gamma_5 \ell) \qquad \lambda_p = V_{pb} V_{pd}^*$$

Effective theory for B decays

- Generic structure of amplitude for B decays

$$\mathcal{A}(\bar{B} \to f) = \sum_i \left[\lambda_{\rm CKM} \times C \times \langle f|\mathcal{O}|\bar{B}\rangle_{\rm QCD+QED}\right]_i$$

- Interplay between
  - Wilson coefficients C , known to NNLL in SM, values at scale $\mu \sim m_b$
    [Bobeth,Misiak,Urban'99;Misiak,Steinhauser'04,Gorbahn,Haisch'04;Gorbahn,Haisch,Misiak'05;Czakon,Haisch,Misiak'06]

$$C_1 = -0.25 \qquad |C_{3,5,6}| < 0.01 \qquad C_7 = -0.30 \qquad C_9 = 4.06$$
$$C_2 = 1.01 \qquad C_4 = -0.08 \qquad C_8 = -0.15 \qquad C_{10} = -4.29$$

  - CKM factors $\lambda_{\rm CKM}$. Hierarchy of CKM elements, weak phase
  - Hadronic matrix elements $\langle f|\mathcal{O}|\bar{B}\rangle$. Can contain strong phases.
- Interplay offers rich and interesting phenomenology for B decays
  - Plethora of data, numerous observables
  - Test of CKM mechanism and indirect search for New Physics
- BUT: Challenging QCD dynamics in hadronic matrix elements.
  Effects from many different scales !!

Inclusive B decays, generalities

- Main tool for inclusive decays: Heavy Quark Expansion

[Khoze,Shifman,Voloshin,Bigi,Uraltsev,Vainshtein,Blok,Chay,Georgi,Grinstein,Luke,...'80s and '90s]

$$\Gamma(B_q \to X) = \frac{1}{2m_{B_q}} \sum_X \int_{\rm PS} (2\pi)^4 \delta^{(4)}(p_{B_q} - p_X) |\langle X|\hat{\mathcal{H}}_{eff}|B_q\rangle|^2$$

- Use optical theorem

$$\Gamma(B_q \to X) = \frac{1}{2m_{B_q}} \langle B_q|\hat{\mathcal{T}}|B_q\rangle \quad \text{with} \quad \hat{\mathcal{T}} = \text{Im}\, i \int d^4x \hat{T}\left[\hat{\mathcal{H}}_{eff}(x)\hat{\mathcal{H}}_{eff}(0)\right]$$

- Expand non-local double insertion of effective Hamiltonian in local operators

$$\begin{aligned}\Gamma &= \Gamma_0 \langle O_{D=3}\rangle + \Gamma_2 \frac{\langle O_{D=5}\rangle}{m_b^2} + \tilde{\Gamma}_3 \frac{\langle \tilde{O}_{D=6}\rangle}{m_b^3} + \ldots \\ &\quad +16\pi^2 \left[\Gamma_3 \frac{\langle O_{D=6}\rangle}{m_b^3} + \Gamma_4 \frac{\langle O_{D=7}\rangle}{m_b^4} + \Gamma_5 \frac{\langle O_{D=8}\rangle}{m_b^5} + \ldots\right]\end{aligned}$$

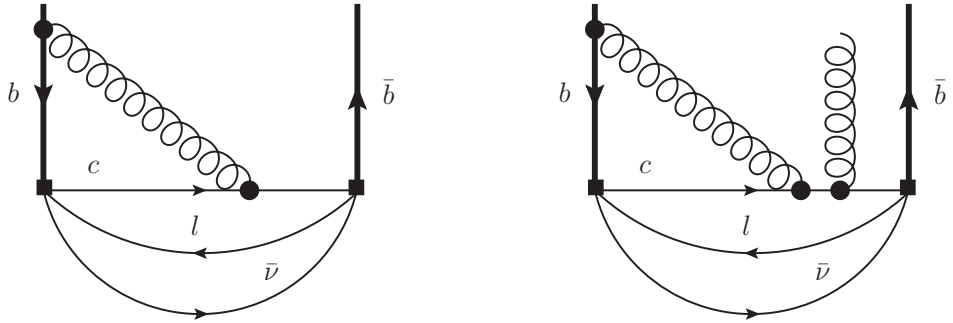
- Expand each term in perturbative series

$$\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_i^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \Gamma_i^{(2)} + \ldots .$$

HQE expansion parameters

- Γ_0: Decay of a free quark, known to $\mathcal{O}(\alpha_s^2)$
- Γ_1: Vanishes due to Heavy Quark Symmetry
- Two terms in Γ_2
  - Kinetic energy μ_π: $2M_B\mu_\pi^2 = -\langle B(v)|\bar{b}_v(iD)^2 b_v|B(v)\rangle$
  - Chromomagnetic moment μ_G: $2M_B\mu_G^2 = -i\langle B(v)|\bar{b}_v\sigma_{\mu\nu}(iD^\mu)(iD^\nu)b_v|B(v)\rangle$
  - Both known to $\mathcal{O}(\alpha_s)$ [Becher,Boos,Lunghi'07;Alberti,Ewerth,Gambino,Nandi'13'14;Mannel,Pivovarov,Rosenthal'15]
- Two more terms in Γ_3
  - Darwin term ρ_D: $2M_H\rho_D^3 = -\langle B(v)|\bar{b}_v(iD_\mu)(ivD)(iD^\mu)b_v|B(v)\rangle$
  - Spin-orbit term ρ_{LS}: $2M_H\rho_{LS}^3 = -i\langle B(v)|\bar{b}_v\sigma_{\mu\nu}(iD^\mu)(ivD)(iD^\nu)b_v|B(v)\rangle$
  - Recently, ρ_D became available at $\mathcal{O}(\alpha_s)$ [Mannel,Pivovarov'19]

Higher orders in HQE and inclusive $B \to X_c \ell \bar{\nu}_\ell$



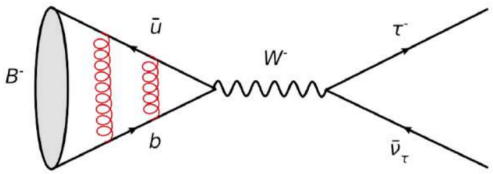
- ρ_D at $\mathcal{O}(\alpha_s)$ [Mannel,Pivovarov'19]

$$\Gamma(B \to X_c \ell \bar{\nu}_\ell) = \Gamma_0 |V_{cb}|^2 \left[a_0 (1 + \frac{\mu_\pi^2}{2m_b^2}) + a_2 \frac{\mu_G^2}{2m_b^2} + \frac{a_D \rho_D + a_{LS} \rho_{LS}}{2m_b^3} + \dots \right]$$

- One finds $a_D = -57.159(1 - \frac{\alpha_s}{4\pi} 6.564) = -57.159(1 - 0.10)$
- Number of parameters grows factorially at higher orders in $1/m$
  - Partial reduction by reparametrisation invariance [Mannel,Vos'18;Fael,Mannel,Vos'19]

Exclusive B decays, generalities

- Leptonic decays



$$\langle 0|\bar{u}\gamma^\mu\gamma_5 b|B^-(p)\rangle = i\, f_B\, p^\mu$$

Exclusive B decays, generalities

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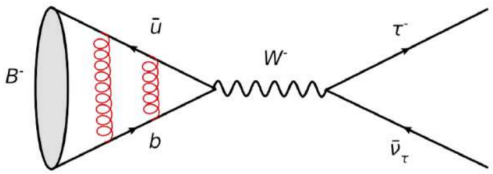

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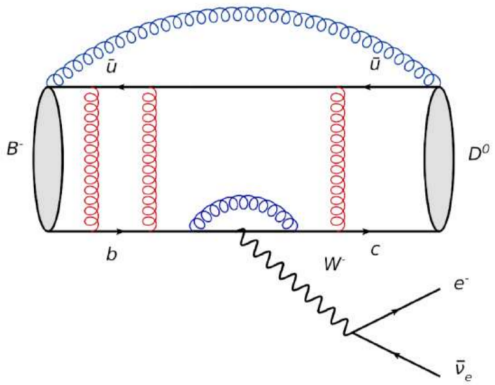
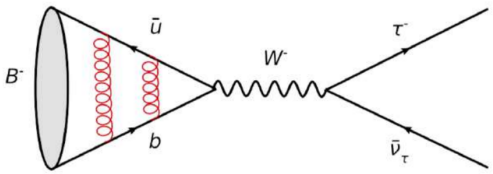
- Semi-leptonic decays



$$\langle D(p')|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle$$
$$= F_+(q^2)\left[(p+p')^\mu - \frac{m_B^2 - m_D^2}{q^2}\, q^\mu\right]$$
$$+ F_0(q^2)\,\frac{m_B^2 - m_D^2}{q^2}\, q^\mu$$

Exclusive B decays, generalities

- Leptonic decays



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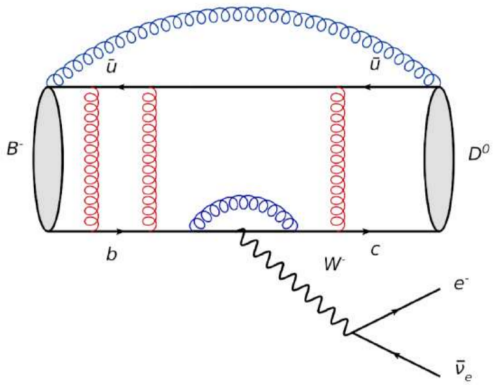

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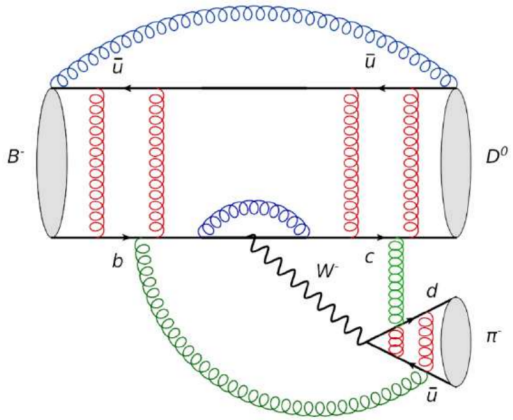
- Non-leptonic decays



$$\langle \pi^- D^+|Q_i|\bar{B}\rangle \simeq m_B^2\, f_{M_2}\, F_+^{B\to D}(m_\pi^2)$$
$$\times \int_0^1 du\;\; T_i^I(u)\, \phi_\pi(u)$$

Exclusive $B \to D^{(*)}\ell\bar{\nu}_\ell$ decays

- Decay rates in terms of form factors $(w = v_B \cdot v_{D^{(*)}})$ [Neubert'91'94]

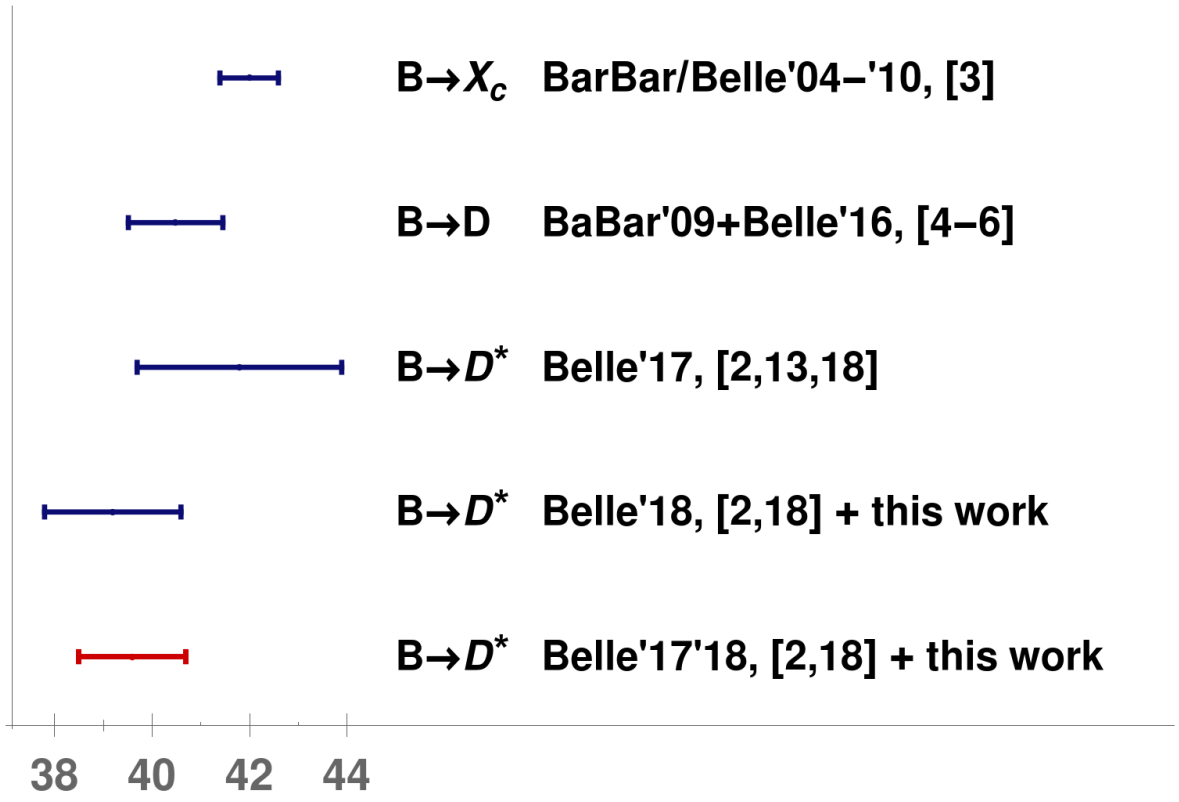
$$\frac{\mathrm{d}\Gamma(\bar{B} \to D^*\ell\,\bar{\nu})}{\mathrm{d}w} = \frac{G_F^2|V_{cb}|^2}{48\pi^3}\,(m_B - m_{D^*})^2\, m_{D^*}^3\,\sqrt{w^2-1}\,(w+1)^2 \times\left[1 + \frac{4w}{w+1}\,\frac{m_B^2 - 2w\,m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\right]|\mathcal{F}(w)|^2$$

$$\frac{\mathrm{d}\Gamma(\bar{B} \to D\,\ell\,\bar{\nu})}{\mathrm{d}w} = \frac{G_F^2|V_{cb}|^2}{48\pi^3}\,(m_B + m_D)^2\, m_D^3 (w^2-1)^{3/2}|V_1(w)|^2$$

- Form factor parametrisations
  - CLN: In terms of an intercept and a slope [Caprini,Lellouch,Neubert'97]
  - BGL: More general, based on dispersion relations, analyticity, and crossing symmetry [Boyd,Grinstein,Lebed'94'95'97]

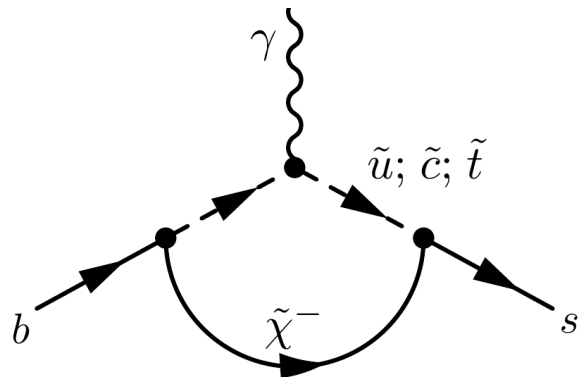
Inclusive vs. exclusive $|V_{cb}|$

- Recent result from global fit [Gambino,Jung,Schacht'19]
  - Difference is currently at the 1.9σ level



Inclusive $\bar{B} \to X_s\gamma$

- One of the standard candles in the search for NP in the quark flavour sector
- Flavour-changing neutral current process
- Dominant contribution is loop-induced
  - Indirectly sensitive to new particles
- Plays a prominent role in global fits



- Current SM prediction vs. measurement (for $E_\gamma > 1.6$ GeV)

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.40 \pm 0.17) \times 10^{-4}$$ [Misiak,Rehman,Steinhauser'20]

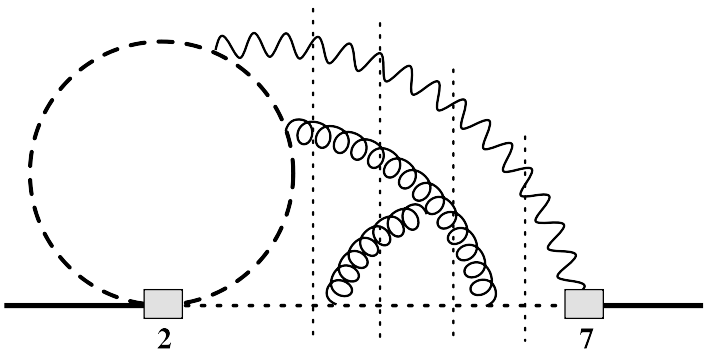
$$\mathcal{B}_{s\gamma}^{\rm exp.} = (3.32 \pm 0.15) \times 10^{-4}$$ [HFLAV'19]

- SM prediction is based on the formula

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = \mathcal{B}(\bar{B} \to X_c\ell\bar{\nu}) \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha_{em}}{\pi\, C} \left[P(E_0) + N(E_0)\right],$$

Perturbative corrections

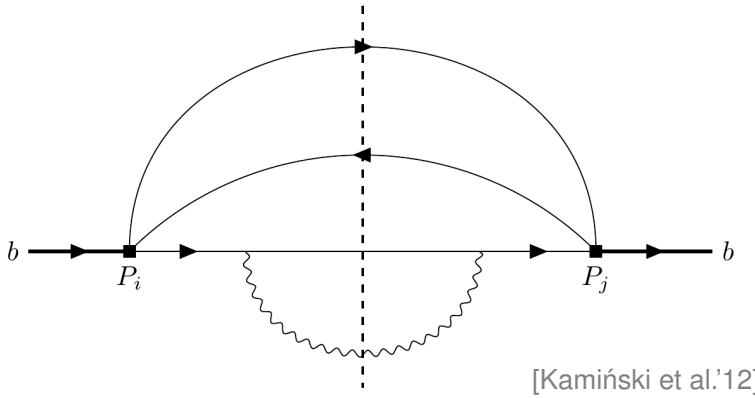
- Perturbative corrections to partonic decay width far advanced
- Many corrections through to NNLO known
- One of the largest uncertainties ($\sim 3\%$) comes from interpolation in m_c in $Q_{1,2} - Q_7$ interference
- Currently underway
  - Exact charm-mass dependence of $Q_{1,2} - Q_7$ interference at NNLO
  - One-loop multi-body contributions, formally NLO but suppressed



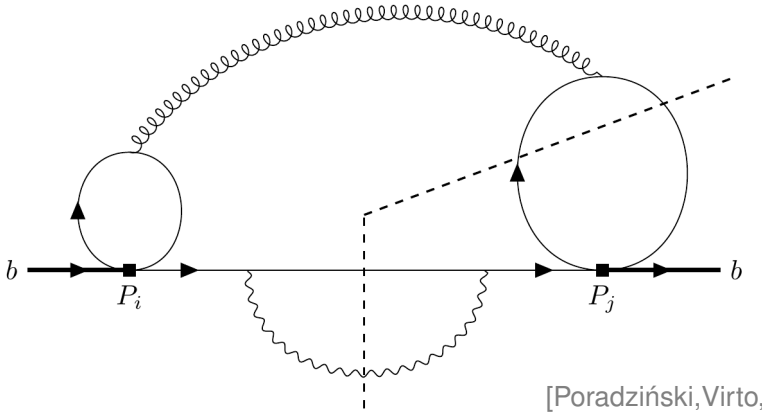
- Large m_c expansion [Misiak,Steinhauser'06]
- Calculation f. $m_c = 0$ [Misiak,Steinhauser,Czakon,TH,et al.'15]
- Exact m_c dependence of fermionic part [Misiak,Rehman,Steinhauser'20]
- Exact m_c dependence of full result [Misiak,Steinhauser,TH,et al.,w.i.p.]

Perturbative corrections

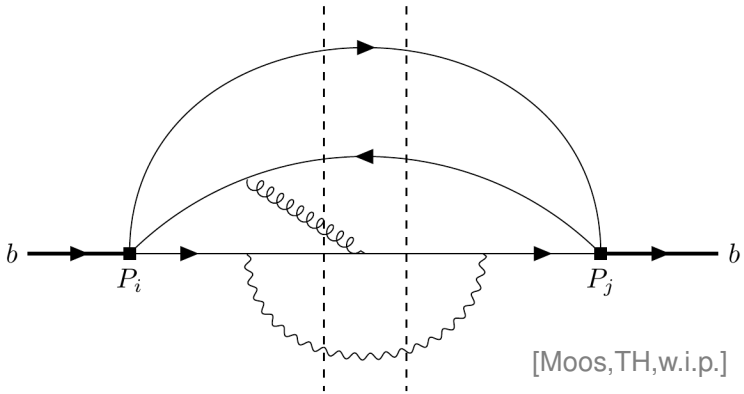
- One-loop multi-body contributions to partonic process $b \to s\, q\bar{q}\, \gamma$
- Formally NLO but suppressed by CKM factors or small Wilson coefficients



[Kamiński et al.'12]



[Poradziński,Virto,TH'14]



[Moos,TH,w.i.p.]

- One-loop four-particle cuts
- Tree-level five-particle cuts
- IBP reduction (reversed unitarity)
- Phase space integrations
- See Lars Moos' talk at YSF

Nonperturbative quantities

[Misiak,Rehman,Steinhauser'20]

- Semileptonic phase space factor

$$
\begin{aligned}
C &= \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B} \to X_c e \bar{\nu}]}{\Gamma[\bar{B} \to X_u e \bar{\nu}]} \\
&= g(z)\ \{0.903 - 0.588\,[\alpha_s(4.6\,\mathrm{GeV}) - 0.22] + 0.0650\,[m_{b,\mathrm{kin}} - 4.55] \\
&- 0.1080\,[m_c(2\,\mathrm{GeV}) - 1.05] - 0.0122\,\mu_G^2 - 0.199\,\rho_D^3 + 0.004\,\rho_{LS}^3\}
\end{aligned}
$$

  - Determined by using HQET methods from measurements of $\bar{B} \to X_c \ell \bar{\nu}$ decay spectra [Alberti,Gambino,Healey,Nandi'14]

Nonperturbative quantities

[Misiak,Rehman,Steinhauser'20]

- Semileptonic phase space factor

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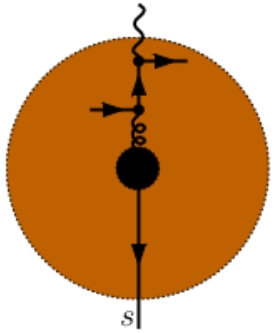
[Alberti,Gambino,Healey,Nandi'14]

- Gluon-photon conversion in $Q_7 - Q_8$ interference



$$\Gamma[B^-\to X_s\gamma] \simeq A + BQ_u + CQ_d + DQ_s,$$
$$\Gamma[\bar{B}^0\to X_s\gamma] \simeq A + BQ_d + CQ_u + DQ_s$$

$$\frac{\delta\Gamma_c}{\Gamma} \simeq \frac{Q_u+Q_d}{Q_d-Q_u}\left[1+2\,\frac{D-C}{C-B}\right]\Delta_{0-} = -\frac{1}{3}(1\pm 0.3)\Delta_{0-} = (0.16\pm 0.74)\%$$

  - Δ_{0-}: Isospin asymmetry, measured at Belle
  - $C - D$ vanishes in isospin limit, assume 30% SU(3) breaking

Nonperturbative quantities

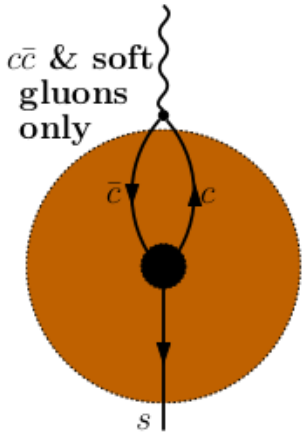
- Resolved photon contribution in $Q_{1,2} - Q_7$ interference

[pics from Misiak'09]

[Voloshin'96;Buchalla,Isidori,Rey'97;Benzke,Hurth,Fickinger,Turczyk'17'20;Gunawardana,Paz'19]

$$N(E_0) \sim C_7 \left(C_2 - \frac{1}{6}C_1\right) \left(-\frac{\mu_G^2}{27m_c^2} + \frac{\Lambda_{17}}{m_b}\right) \equiv \delta N_V \, \kappa_V$$

$$\Lambda_{17} = e_c \,\mathrm{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2 - i\varepsilon}{m_b\,\omega_1}\right) + \frac{m_b\,\omega_1}{12m_c^2}\right] h_{17}(\omega_1, \mu)$$



- h_{17}: soft function, modeled in [Gunawardana,Paz'19]
- Yields $\Lambda_{17} \in [-24, 5]$ MeV $\implies$ $\kappa_V = 1.2 \pm 0.3$

[see also Benzke,Lee,Neubert,Paz'10;Benzke,Hurth'20]

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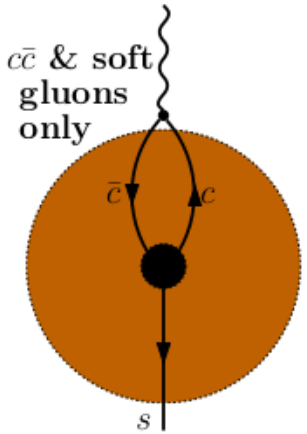

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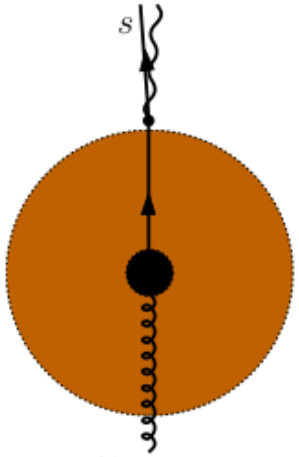
[see also Benzke,Lee,Neubert,Paz'10;Benzke,Hurth'20]

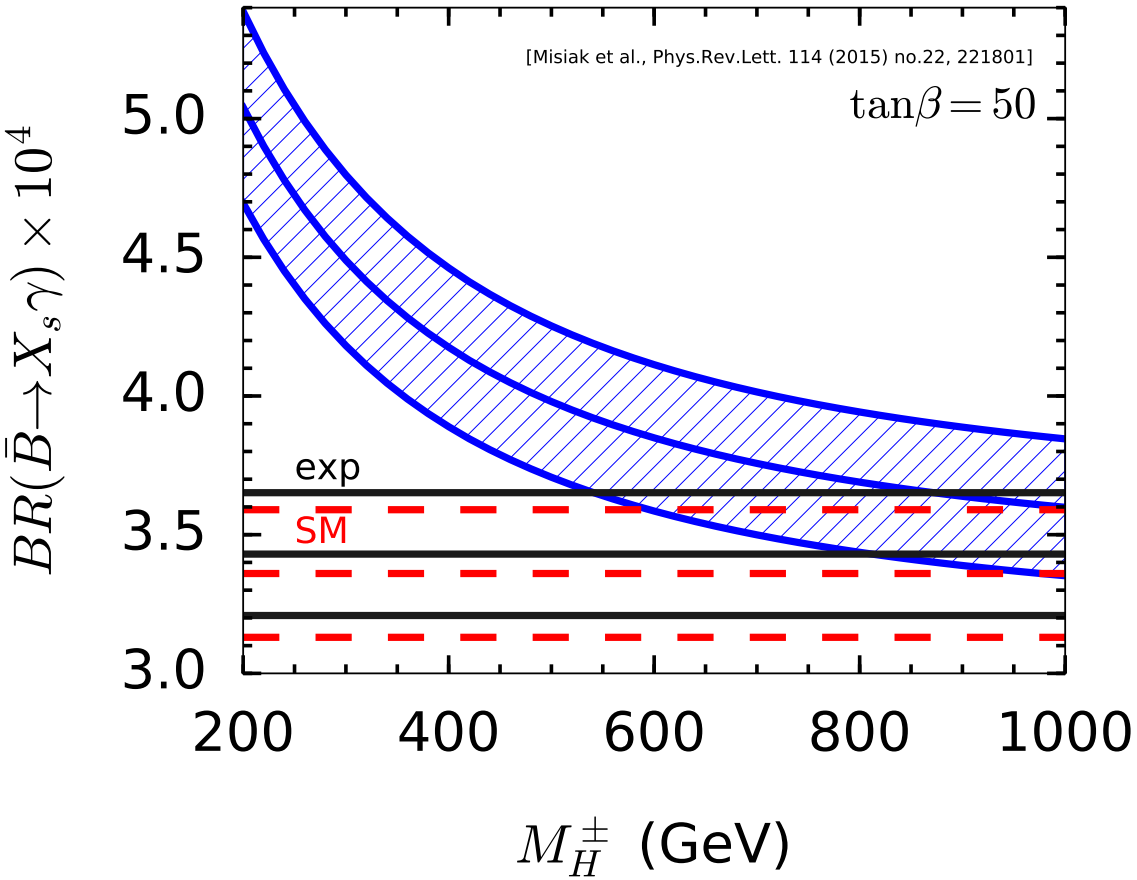
- Gluon-photon conversion in $Q_7 - Q_8$ interference
  - $P(E_0)$ receives corrections $\propto |C_8|^2 \log(m_b/m_s)$
  - Small contribution (<1%), but large uncertainty
  - Vary $\log(m_b/m_s) \in [\log(10), \log(50)]$ [Czakon et al.'15]
  - Additional nonperturbative effects $\propto |C_8|^2$ impact $\mathcal{B}_{s\gamma}$ in range $[-0.3, 1.9]$% [Benzke,Lee,Neubert,Paz'10]
  - Reproduce numerically by

    $\log(m_b/m_s) \to \kappa_{88} \log(50)$ and $\kappa_{88} = 1.7 \pm 1.1$



NP constraints

- Charged Higgs mass in two-Higgs doublet models



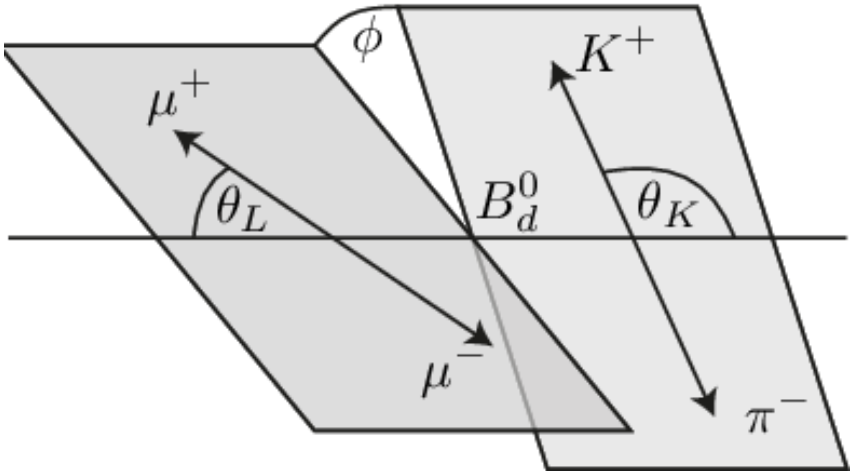
- Updates in 2017 and 2020. Latest bound: $M_{H+} > 800$ GeV at 95% C.L.

[Misiak,Rehman,Steinhauser'20]

Exclusive $\bar{B} \to K^* \ell^+ \ell^-$

- Angular analysis of $\bar{B} \to K^*(\to K\pi)\ell^+\ell^-$

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_K\, d\cos\theta_l\, d\phi} = \frac{9}{32\pi}\Big[J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l$$
$$+J_3\sin^2\theta_K\sin^2\theta_l\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_l\cos\phi + J_5\sin 2\theta_K\sin\theta_l\cos\phi$$
$$+(J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l + J_7\sin 2\theta_K\sin\theta_l\sin\phi + J_8\sin 2\theta_K\sin 2\theta_l\sin\phi$$
$$+J_9\sin^2\theta_K\sin^2\theta_l\sin 2\phi\Big]$$



- Construct th. + exptl. robust observables

$$P_5' = \frac{J_5}{2\sqrt{-J_{2c}J_{2s}}} = \sqrt{2}\,\frac{\mathrm{Re}(\mathcal{A}_0^L\mathcal{A}_\perp^{L*} - \mathcal{A}_0^R\mathcal{A}_\perp^{R*})}{\sqrt{|\mathcal{A}_0|^2}\sqrt{|\mathcal{A}_\parallel|^2 + |\mathcal{A}_\perp|^2}}$$

[Descotes-Genon,Matias,Mescia,Ramon,Virto'12]

Exclusive $\bar{B} \to K^* \ell^+ \ell^-$

- Transversity amplitudes ($\lambda = \perp, \|, 0$)

$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- Local Form factors: $\mathcal{F}_\lambda^{(T)}(q^2) = \langle K_\lambda^*(k) | \bar{s}\, \Gamma_\lambda^{(T)}\, b | \bar{B}(k+q) \rangle$

- Non-local FFs: $\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x\, e^{iq\cdot x} \langle K_\lambda^*(k) | T\{ \mathcal{J}_{\rm em}^\mu(x), C_i Q_i(0) \} | \bar{B}(k+q) \rangle$

Exclusive $\bar{B} \to K^* \ell^+ \ell^-$

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$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

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- Non-local FFs: $\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x\, e^{iq\cdot x} \langle K_\lambda^*(k) | T\{ \mathcal{J}_{\rm em}^\mu(x), C_i Q_i(0) \} | \bar{B}(k+q) \rangle$

- In the heavy-quark limit and at large recoil (large E_{K^*}, low q^2) the seven $B \to K^*$ form factors reduce to two universal (soft) form factors.

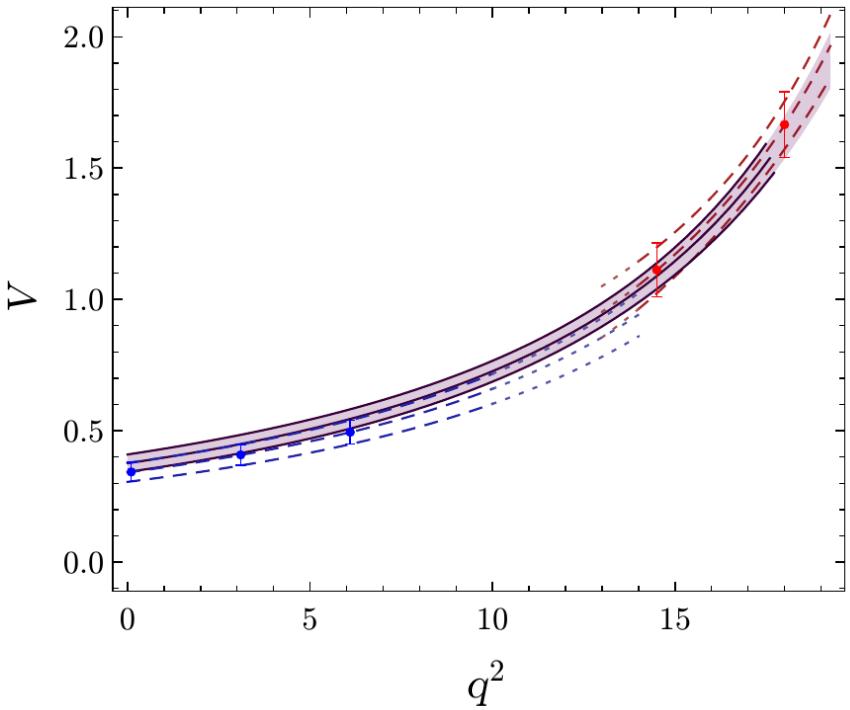
[Beneke,Feldmann'00;Beneke,Feldmann,Seidel'01'04]

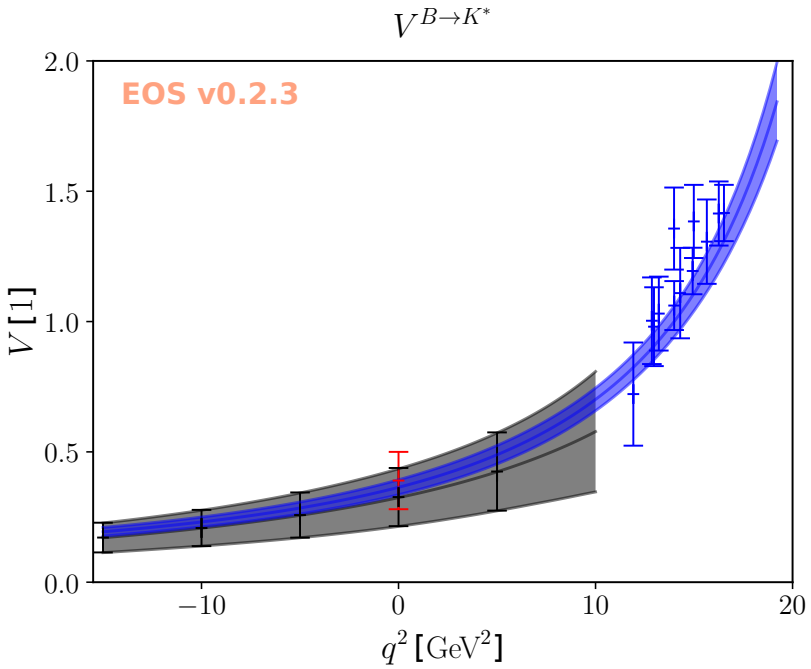
$$V(q^2) = \frac{m_B + m_{K^*}}{m_B} \xi_\perp(q^2) \qquad A_0(q^2) = \frac{E_K^*}{m_{K^*}} \xi_\|(q^2)$$

  - Soft form factors cancel in specific angular observables, which then depend on short-distance information only (e.g. $A_T^{(1)}$, $A_T^{(2)}$, P_5', ...)

Exclusive $\bar{B} \to K^* \ell^+ \ell^-$

- Local form factors from sum rules (at low q^2) and lattice QCD (at high q^2)
  - For example $V(q^2) \simeq \mathcal{F}_\perp^{BK^*}(q^2)$





sum rules [Bharucha,Straub,Zwicky'15] [Gubernari,Kokulu,van Dyk'18]

lattice QCD [Horgan,Liu,Meinel,Wingate'13] [Horgan,Liu,Meinel,Wingate'13'15]

Exclusive $\bar{B} \to K^* \ell^+ \ell^-$

- Non-local form factors: OPE + dispersion relation

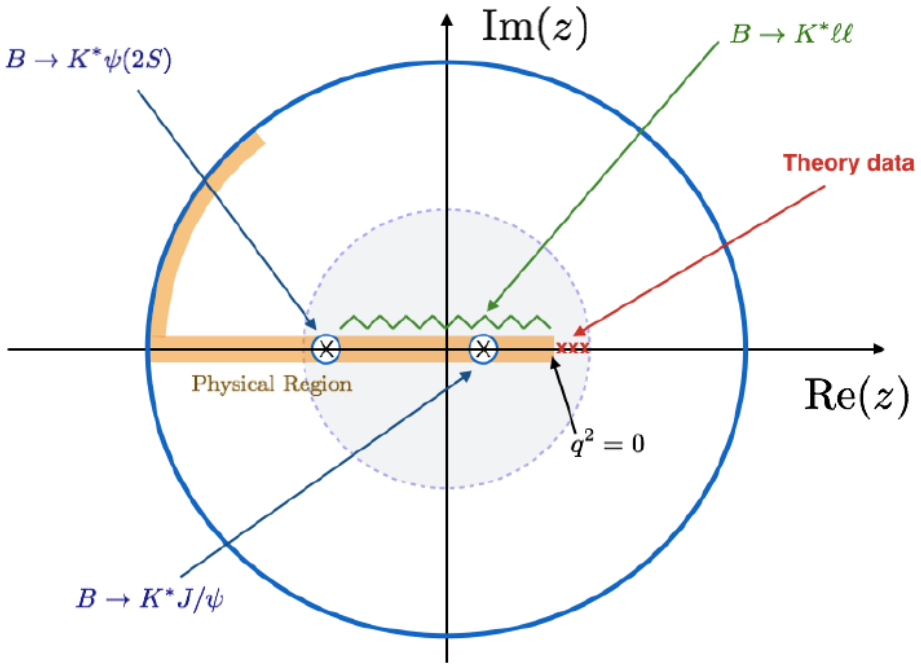
$$\mathcal{H}_{\lambda,x}(q^2) = \mathcal{H}^{\rm OPE}_{\lambda,x}(q_0^2 < 0) + (q^2 - q_0^2) \int_{s_{\rm thr.}}^{\infty} dt \, \frac{\rho_{\lambda,x}(t)}{(t - q^2 - i\eta)(t - q_0^2)}$$

- $\mathcal{H}^{\rm OPE}_{\lambda,x}(q_0^2 < 0)$ from theory [Khodjamirian,Mannel,Pivovarov,Wang'10'12]
- Transition from OPE region to physical region requires data on $B \to K^{(*)} X_{1--}$
  - $\rho_{\lambda,c}(t)$: $B \to K^* J/\psi$, $B \to K^* \psi(2S)$, $B \to K^* D\bar{D}$, ...
    - Charm contribution numerically leading
  - $\rho_{\lambda,s}(t)$: $B \to K^* \phi$, $B \to K^* K\bar{K}$, ...
  - $\rho_{\lambda,ud}(t)$: $B \to K^* \rho$, $B \to K^* \omega$, $B \to K^* \pi\pi$, ...

Exclusive $\bar{B} \to K^* \ell^+ \ell^-$

[Bobeth,Chrzaszcz,van Dyk,Virto'17]

- Non-local form factors



Constrain non-local effect with $B \to K^* \phi_n$

Use interresonance $B \to K^* \ell\ell$ DATA

- Use conformal variable

$$z(q^2) \equiv \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_+ = 4M_D^2$$

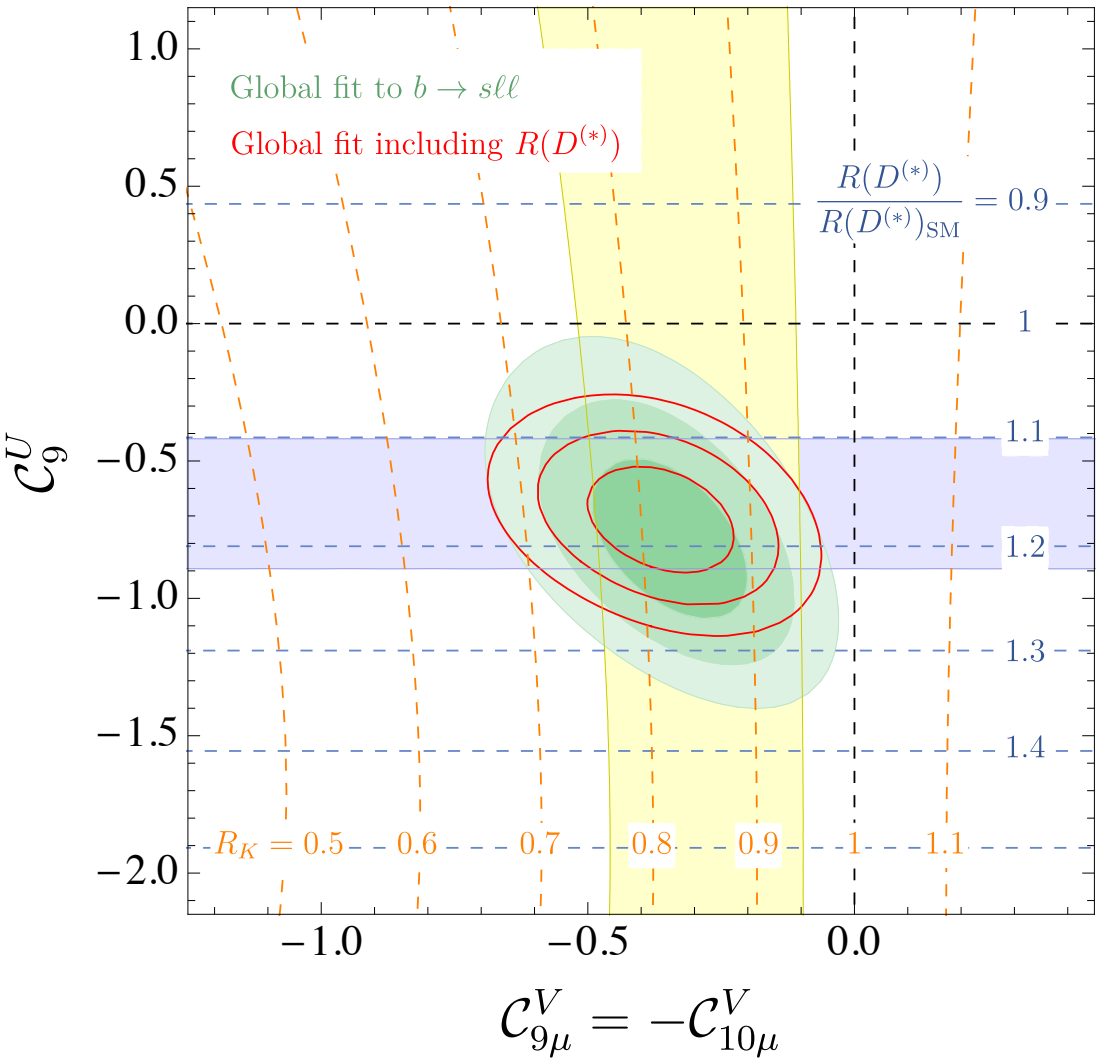
$$t_0 = t_+ - \sqrt{t_+(t_+ - M^2_{\psi(2S)})}$$

- Maps 1st Riemann sheet in q^2 onto $|z| < 1$
- Maps $c\bar{c}$ branch cut in q^2 onto $|z| = 1$
- Maps -7 GeV$^2 < q^2 < M^2_{\psi(2S)}$ onto $|z| < 0.52$

Exclusive $\bar{B} \to K^* \ell^+ \ell^-$

[Alguero et al.'19'20]

- Global fits to exclusive $b \to s\ell\ell$



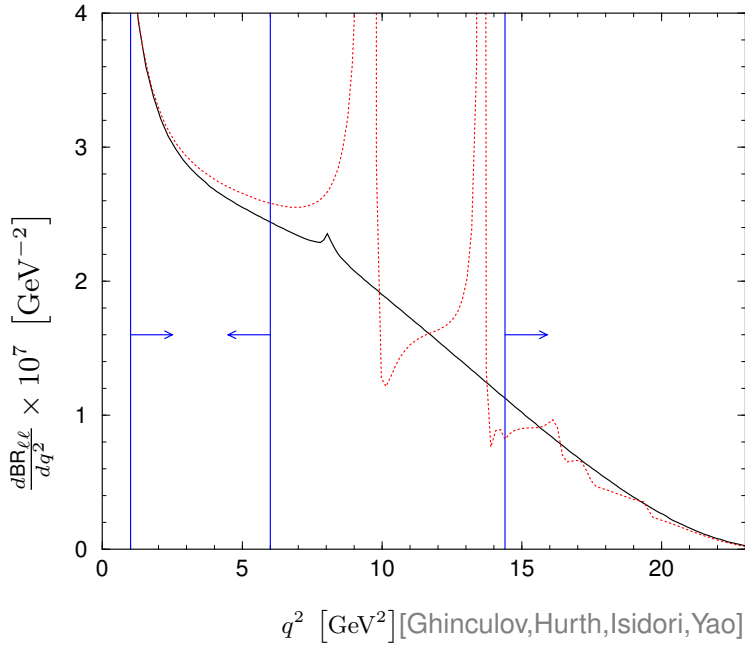
- $C_i^e = C_i^U$
- $C_i^\mu = C_i^U + C_i^V$

Inclusive $\bar{B} \to X_s \ell^+ \ell^-$

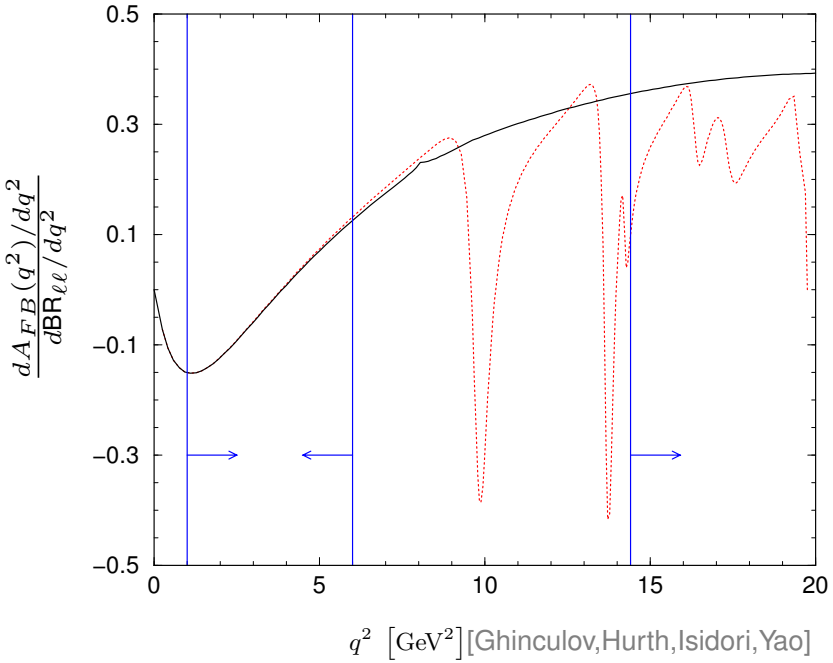
- Double differential decay width ($z = \cos\theta_\ell$) [Lee,Ligeti,Stewart,Tackmann'06]

$$\frac{d^2\Gamma}{dq^2\,dz} = \frac{3}{8}\left[(1+z^2)\,H_T(q^2) + 2\,z\,H_A(q^2) + 2\,(1-z^2)\,H_L(q^2)\right]$$

Note: $$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2)\,, \qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\,H_A(q^2)$$



[Ghinculov,Hurth,Isidori,Yao]



[Ghinculov,Hurth,Isidori,Yao]

- Low-q^2 region: $1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2$
- High-q^2 region: $q^2 > 14.4\,\mathrm{GeV}^2$

Inclusive $\bar{B} \to X_s \ell^+ \ell^-$

- Dependence of the H_i on Wilson coefficients ($s = q^2/m_b^2$)

$$H_T(q^2) \propto 2s(1-s)^2 \left[\left| C_9 + \frac{2}{s} C_7 \right|^2 + |C_{10}|^2 \right]$$
$$H_A(q^2) \propto -4s(1-s)^2 \ \mathrm{Re}\left[C_{10} \left(C_9 + \frac{2}{s} C_7 \right) \right]$$
$$H_L(q^2) \propto (1-s)^2 \left[\left| C_9 + 2\, C_7 \right|^2 + |C_{10}|^2 \right]$$

- Normalisation $$\frac{d\,\mathcal{B}(\bar{B} \to X_s ll)}{d\,\hat{s}} = \mathcal{B}^{exp.}_{b \to c\,e\,\nu} \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{1}{C} \frac{d\Gamma(\bar{B} \to X_s\, ll)/d\hat{s}}{\Gamma(\bar{B} \to X_u e \bar{\nu})}$$

- LFU ratio $$R_{X_s}[q_m^2, q_M^2] \equiv \int_{q_m^2}^{q_M^2} dq^2 \frac{d\Gamma_{\ell=\mu}}{dq^2} \Big/ \int_{q_m^2}^{q_M^2} dq^2 \frac{d\Gamma_{\ell=e}}{dq^2}$$

- High-q^2 region, introduce the ratio $$\mathcal{R}(s_0) = \frac{\int_{\hat{s}_0}^1 \mathrm{d}\hat{s} \ \ d\Gamma(\bar{B} \to X_s \ell^+ \ell^-)/d\hat{s}}{\int_{\hat{s}_0}^1 \mathrm{d}\hat{s} \ \ d\Gamma(\bar{B}^0 \to X_u \ell \nu)/d\hat{s}}$$

  [Ligeti,Tackmann'07]

  - Normalize to semileptonic $\bar{B}^0 \to X_u \ell \nu$ rate with the same cut
    Need differential semi-leptonic $b \to u$ rate

Perturbative and non-perturbative corrections

$$\Gamma(\bar{B} \to X_s\, \ell\ell) = \Gamma(b \to X_s\, \ell\ell) \;+\; \text{power corrections}$$

- Pert. corrections at quark level are known to NNLO QCD + NLO QED
  [Misiak,Buras,Münz,Bobeth,Urban,Asatrian,Asatryan,Greub,Walker,Bobeth,Gambino,Gorbahn,Haisch,Blokland]
  [Czarnecki,Melnikov,Slusarczyk,Bieri,Ghinculov,Hurth,Isidori,Yao,Greub,Pilipp,Schüpbach,Lunghi,TH]
  - Fully differential QCD corrections at NNLO for $Q_{9,10}$ also known
    [Brucherseifer,Caola,Melnikov'13]
- $1/m_b^2$, $1/m_b^3$ and $1/m_c^2$ non-pert. corrections [Falk,Luke,Savage'93] [Ali,Hiller,Handoko,Morozumi'96] [Bauer,Burrell'99; Buchalla,Isidori,Rey'97]
- New in 2020 update [Hurth,Jenkins,Lunghi,Qin,Vos,TH'20]
  - SM prediction of all angular observables + LFU ratio R_{X_s}
  - More sophisticated implementation of factorizable $c\bar{c}$ contributions via KS approach [Krüger, Sehgal'96]
  - Resolved contributions from charm loops [Benzke,Hurth,Fickinger,Turczyk'17-'20]
  - Monte Carlo study of collinear photon radiation, tailored for Belle II analysis
  - Comprehensive model-independent new-physics analysis

SM predictions

- Branching ratio low-q^2 region

[Hurth,Jenkins,Lunghi,Qin,Vos,TH'20]

$$\mathcal{B}[1,6]_{\mu\mu} = (17.29 \pm 0.76_{\text{scale}} \pm 0.19_{m_t} \pm 0.39_{C,m_c} \pm 0.20_{m_b} \pm 0.09_{\alpha_s} \pm 0.02_{\text{CKM}} \pm 0.26_{\text{BR}_{\text{sl}}} \pm 0.12_{\lambda_2} \pm 0.86_{\text{resolved}}) \times 10^{-7} = (17.29 \pm 1.28) \times 10^{-7}$$

  - Total error 7.5%, dominated by scale uncertainty and resolved contributions

SM predictions

- Branching ratio low-q^2 region [Hurth,Jenkins,Lunghi,Qin,Vos,TH'20]

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- Ratio R_{X_s} has small uncertainty, $R_{X_s}[1,6] = 0.971 \pm 0.003$

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- Ratio R_{X_s} has small uncertainty, $R_{X_s}[1,6] = 0.971 \pm 0.003$
- Branching ratio, high-q^2 region

$$\mathcal{B}[>14.4]_{\mu\mu} = (2.38 \pm 0.27_{\text{scale}} \pm 0.03_{m_t} \pm 0.04_{C,m_c} \pm 0.21_{m_b} \pm 0.002_{\text{CKM}} \pm 0.04_{\text{BR}_{\text{sl}}} \pm 0.006_{\alpha_s} \pm 0.12_{\lambda_2} \pm 0.57_{\rho_1} \pm 0.54_{f_{u,s}}) \times 10^{-7} = (2.38 \pm 0.87) \times 10^{-7}$$

  - Total error >30%, dominated by HQET annihilation matrix elements

SM predictions

- Branching ratio low-q^2 region [Hurth,Jenkins,Lunghi,Qin,Vos,TH'20]

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  - Total error >30%, dominated by HQET annihilation matrix elements
- Different normalisation

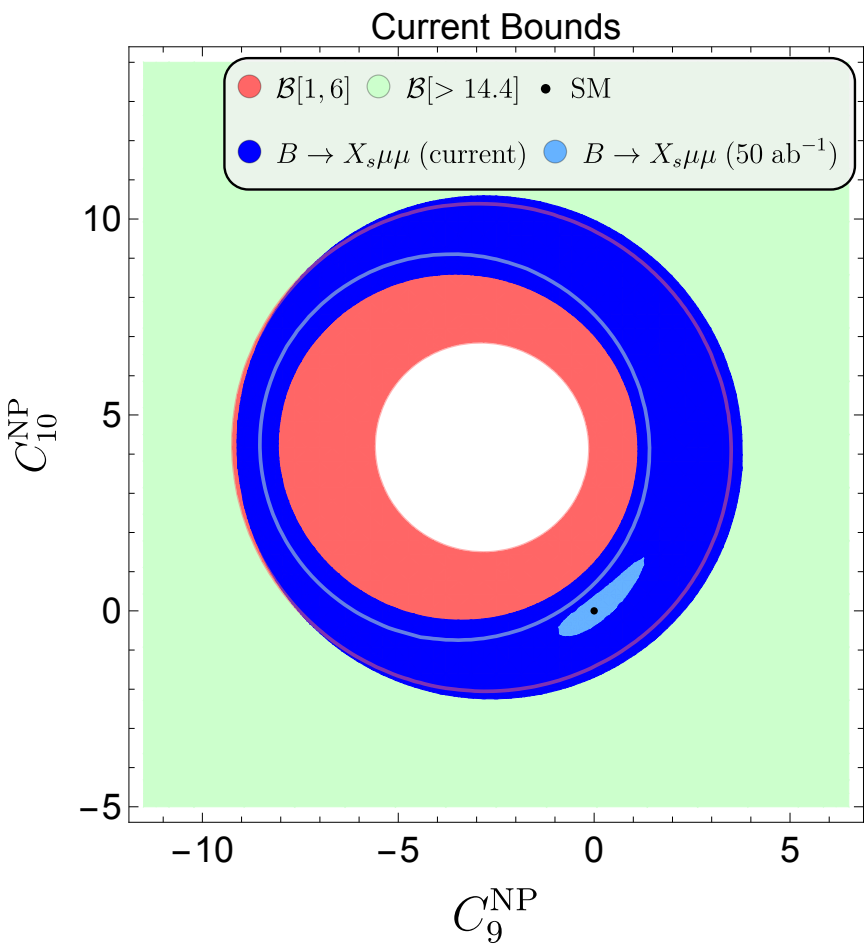
$$\mathcal{R}(14.4)_{\mu\mu} = (25.33 \pm 0.27_{\text{scale}} \pm 0.29_{m_t} \pm 0.14_{C,m_c} \pm 0.03_{m_b} \pm 0.07_{\alpha_s} \pm 1.09_{\text{CKM}} \pm 0.04_{\lambda_2} \pm 0.83_{\rho_1} \pm 1.29_{f_{u,s}}) \times 10^{-4} = (25.33 \pm 1.93) \times 10^{-4}$$

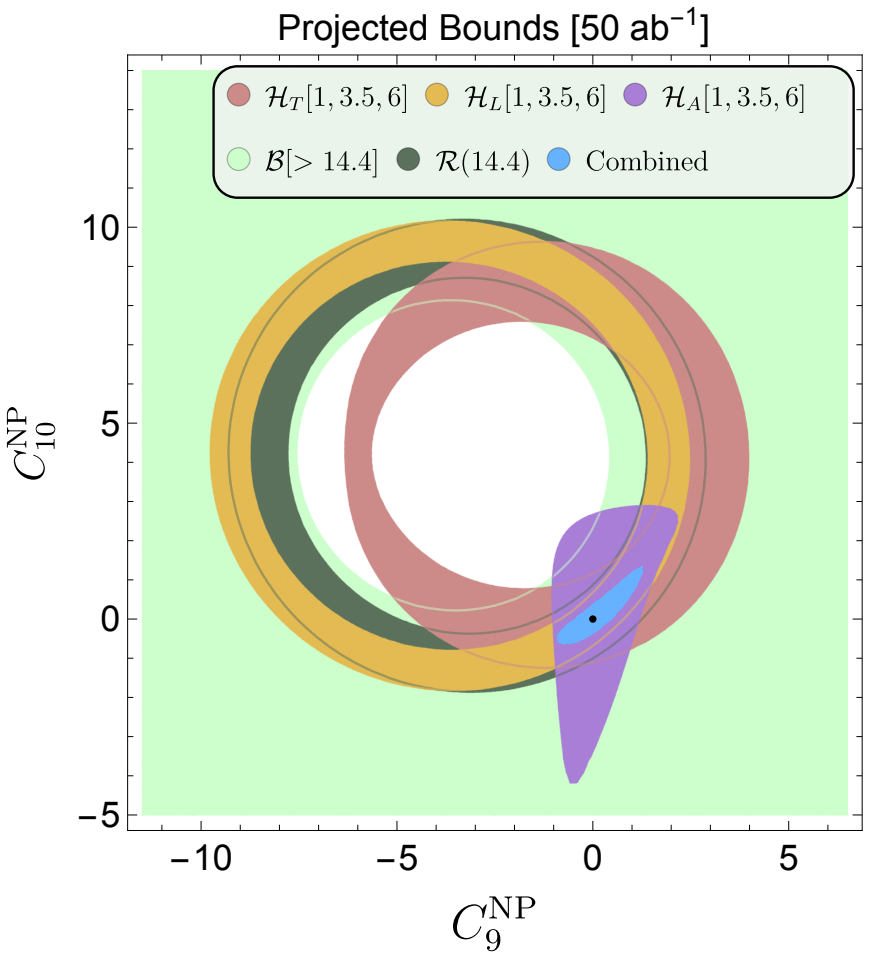
  - Total error <10%

NP constraints

[Hurth,Jenkins,Lunghi,Qin,Vos,TH'20]

- Current and projected bounds from inclusive $\bar{B} \to X_s \ell^+ \ell^-$

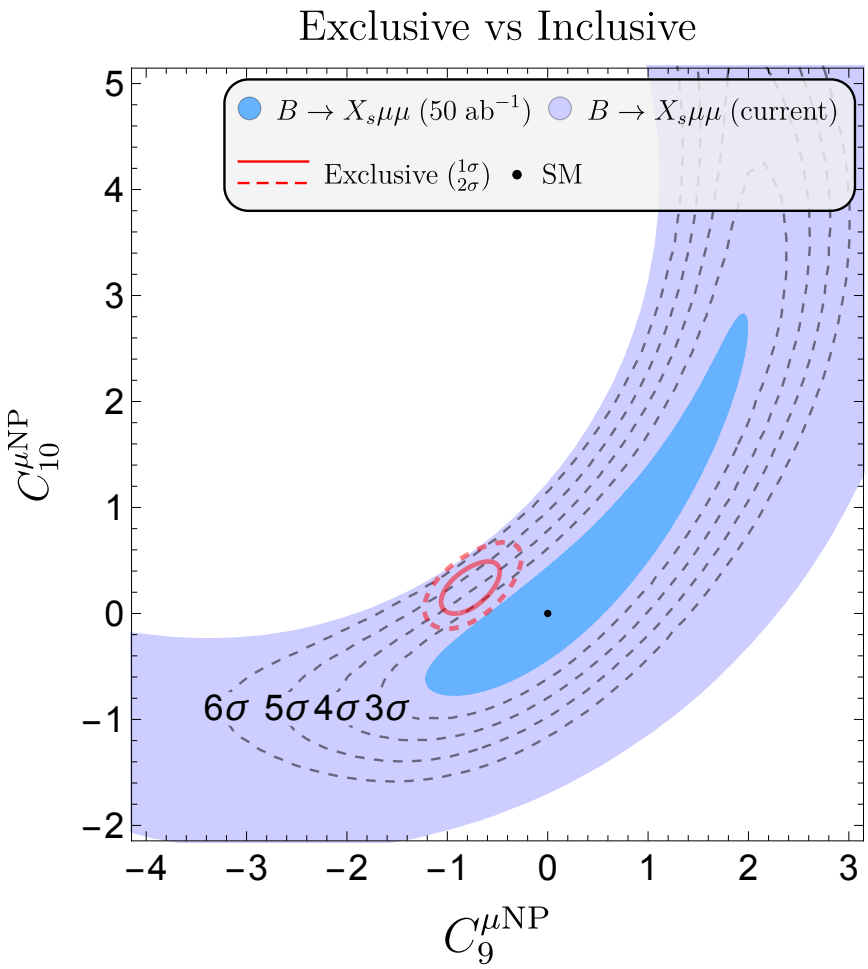


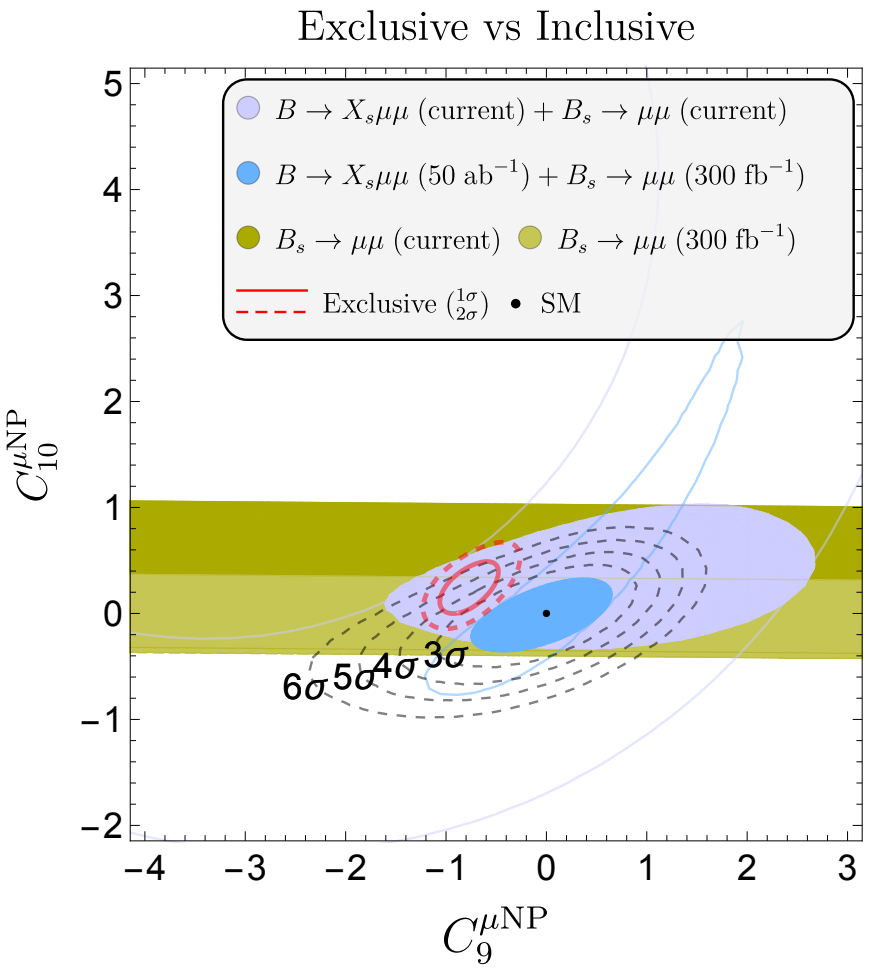


NP constraints

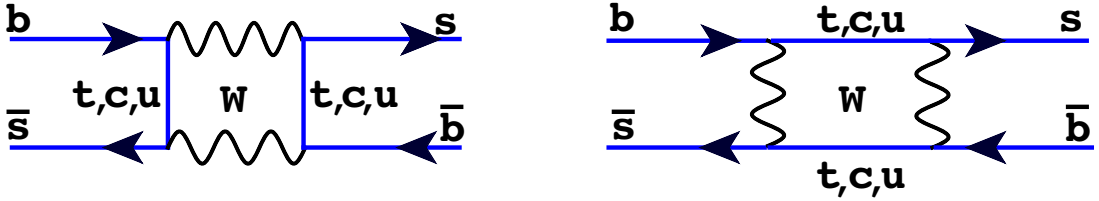
[Hurth,Jenkins,Lunghi,Qin,Vos,TH'20]

- Projected bounds using interplay with exclusive $\bar{B} \to K^{(*)}\ell^+\ell^-$ and $\bar{B}_s \to \mu^+\mu^-$





B meson mixing



$$i\frac{d}{dt}\begin{pmatrix} |B_q\rangle \\ |\bar{B}_q\rangle \end{pmatrix} = \left[\begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{11} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11} \end{pmatrix}\right]\begin{pmatrix} |B_q\rangle \\ |\bar{B}_q\rangle \end{pmatrix} = \left[M - \frac{i}{2}\Gamma\right]\begin{pmatrix} |B_q\rangle \\ |\bar{B}_q\rangle \end{pmatrix}$$

- Diagonalise $M - i\Gamma/2 \quad \Longrightarrow \quad$ eigenvalues $M_H - i\Gamma_H/2$ and $M_L - i\Gamma_L/2$
- Relate $|M_{12}^q|$, $|\Gamma_{12}^q|$ and $\phi^q = \arg(-M_{12}^q/\Gamma_{12}^q)$ to three observables
  - Mass difference: $\Delta M = M_H - M_L \simeq 2\,|M_{12}|$
  - Width difference: $\Delta\Gamma = \Gamma_L - \Gamma_H \simeq 2\,|\Gamma_{12}|\,\cos\phi$
  - Flavour-specific CP asymmetry

$$a_{fs}^q = \frac{\Gamma(\bar{B}_q(t) \to f) - \Gamma(B_q(t) \to \bar{f})}{\Gamma(\bar{B}_q(t) \to f) + \Gamma(B_q(t) \to \bar{f})} = \mathrm{Im}\Big(\frac{\Gamma_{12}^q}{M_{12}^q}\Big) = \left|\frac{\Gamma_{12}^q}{M_{12}^q}\right| \sin\phi^q$$

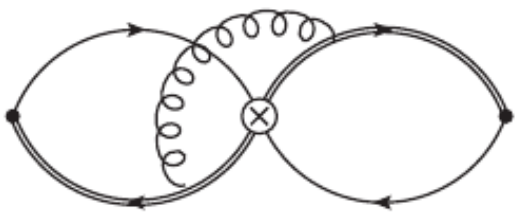
  - a_{fs}^q measured from semileptonic decays, small in SM

Mass difference

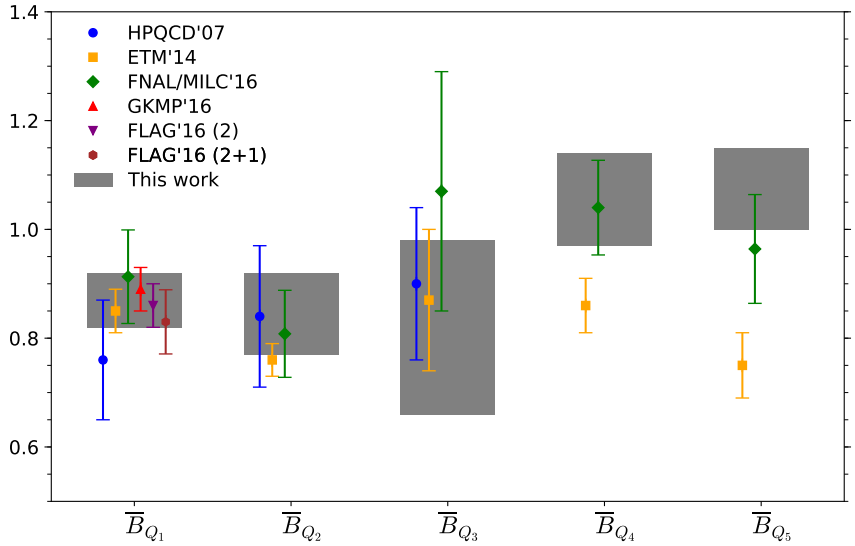
- Many applications
  - CKM elements, $|V_{td}/V_{ts}|$, $|V_{ts}V_{tb}|$, ...
  - UT angles, BSM constraints

- In the SM $$M_{12}^s = \frac{G_F^2}{12\pi^2}\,\lambda_t^2\,M_W^2 S_0(x_t)\,B(\mu)\,f_{B_s}^2\,M_{Bs}\,\hat{\eta}_B$$

- Extraction of decay constant f_{B_s} and bag parameter $B(\mu)$ from
  - Lattice QCD: ETM (2013), FNAL-MILC (2016), HPQCD (2019)
  - HQET Sum rules: Mannel et al. (2016-18), Lenz et al. (2017)



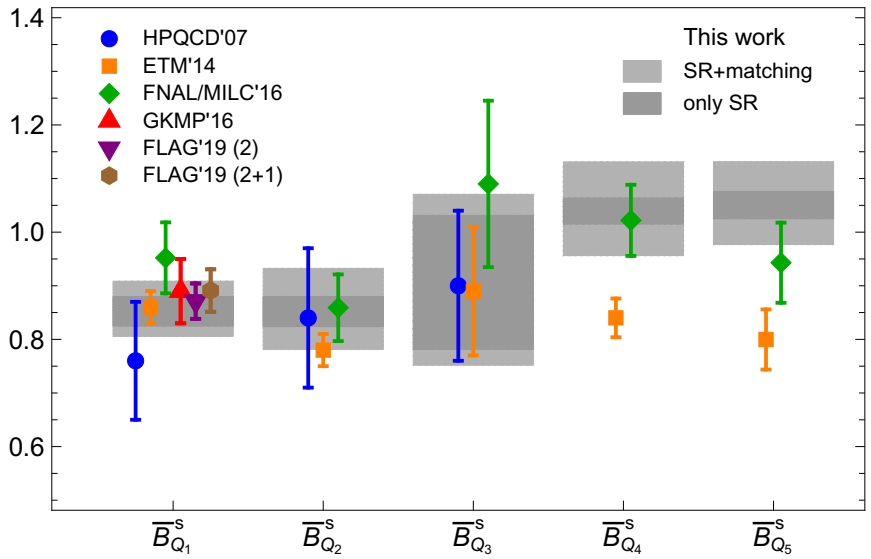
[see also Grozin,Lee'08]



[Kirk,Lenz,Rauh'17]

Mass difference

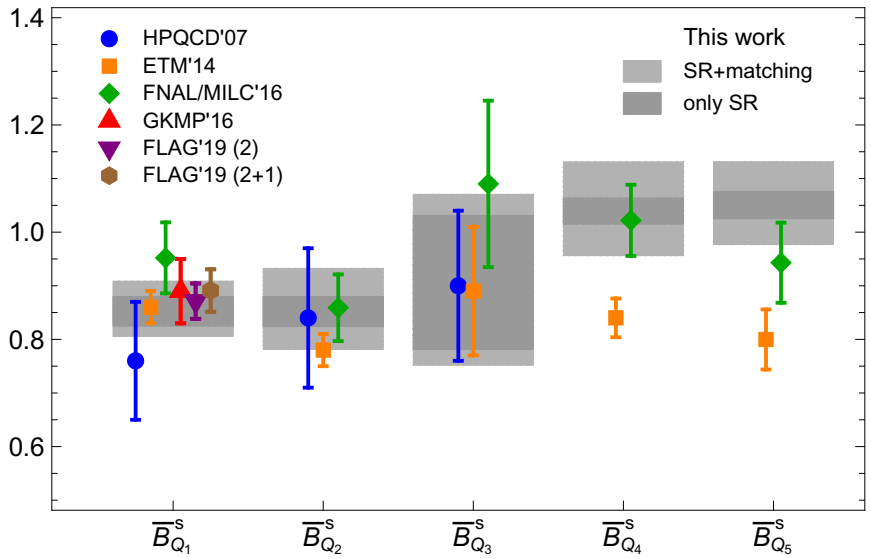
- Same analysis for B_s system



[King,Lenz,Rauh'19]

Mass difference

- Same analysis for B_s system

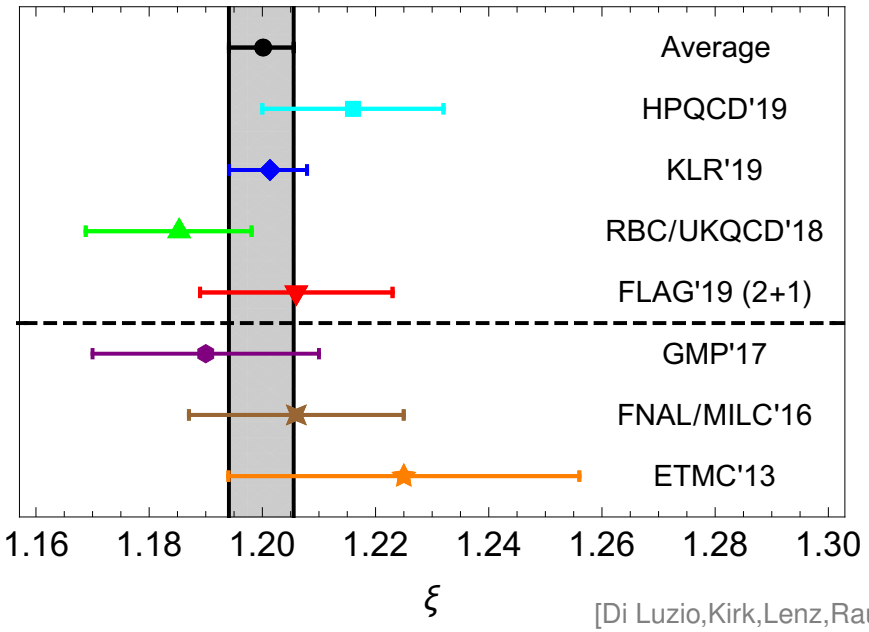


[King,Lenz,Rauh'19]

- Very precise prediction of ratio

[Di Luzio,Kirk,Lenz,Rauh'19]

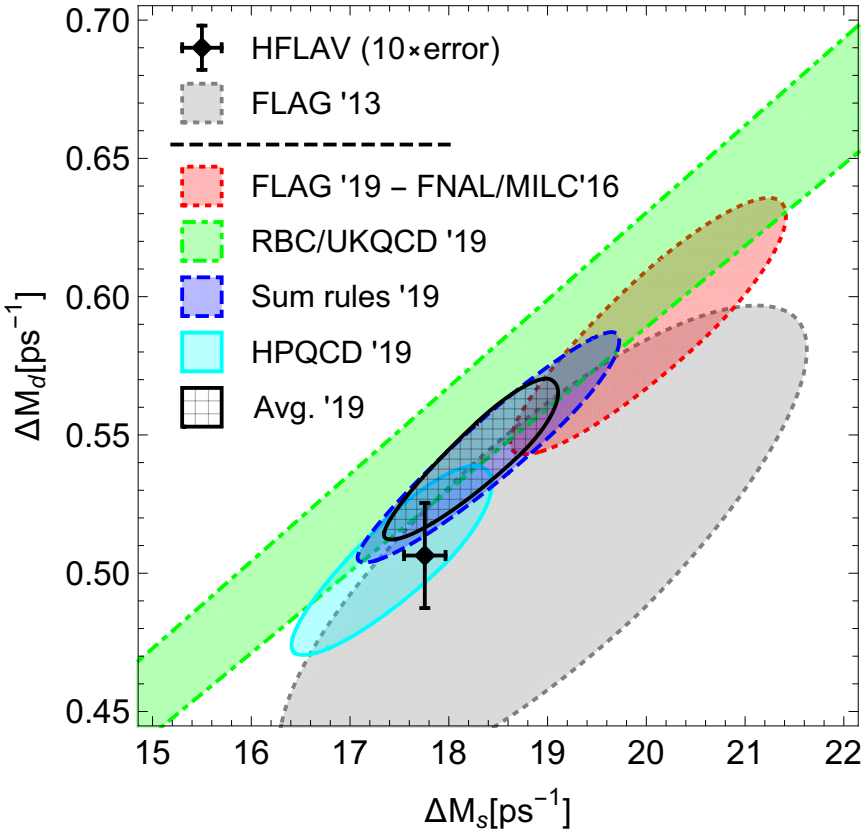
$$\xi = \frac{f_{B_s}}{f_{B_d}} \frac{\sqrt{B_1^{B_s}}}{\sqrt{B_1^{B_d}}} = 1.200^{+0.0054}_{+0.0060}$$



[Di Luzio,Kirk,Lenz,Rauh'19]

Mass difference

[King,Di Luzio,Kirk,Lenz,Rauh'19]



- Average of lattice + sum rules

$$\Delta M_d^{\text{Average 2019}} = \left(0.533^{+0.022}_{-0.036}\right) \text{ps}^{-1} = \left(1.05^{+0.04}_{-0.07}\right) \Delta M_d^{\text{exp}}$$
$$\Delta M_s^{\text{Average 2019}} = \left(18.4^{+0.7}_{-1.2}\right) \text{ps}^{-1} = \left(1.04^{+0.04}_{-0.07}\right) \Delta M_s^{\text{exp}} .$$

- Good agreement with expt.

$$\left|\frac{V_{td}}{V_{ts}}\right| = 0.2043^{+0.0010}_{-0.0011}$$

- Slightly below CKMfits

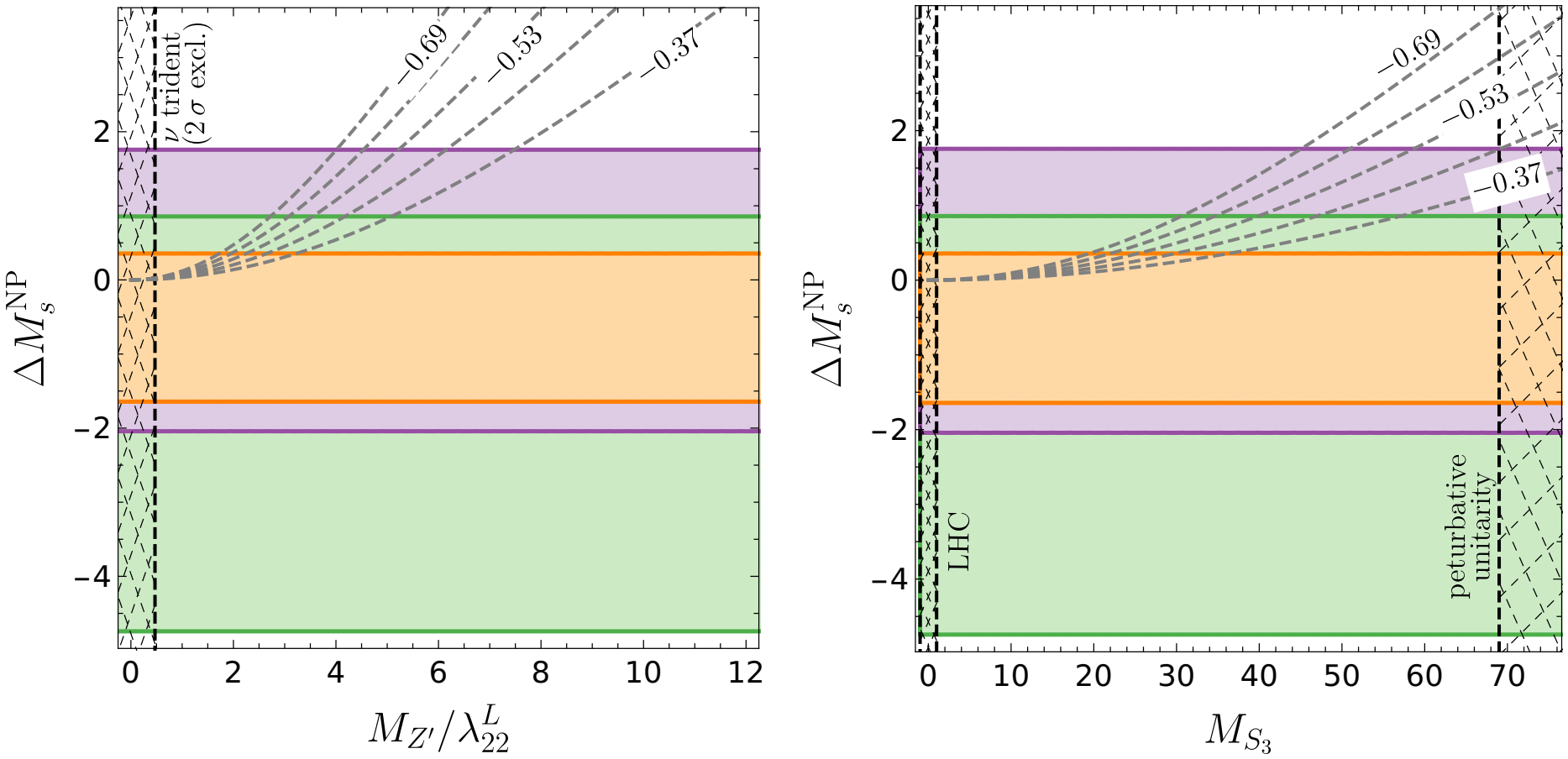
- Prediction for ratio of mass differences

$$\frac{\Delta M_d}{\Delta M_s} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{1}{\xi^2} \frac{M_{B_d}}{M_{B_s}} = 0.0298^{+0.0005}_{-0.0009}$$

- Agreement at 1.4σ with experiment: $(\Delta M_d/\Delta M_s)_{\text{exp.}} = 0.0285 \pm 0.0001$

Mass differences and New Physics

Current bounds and future projections

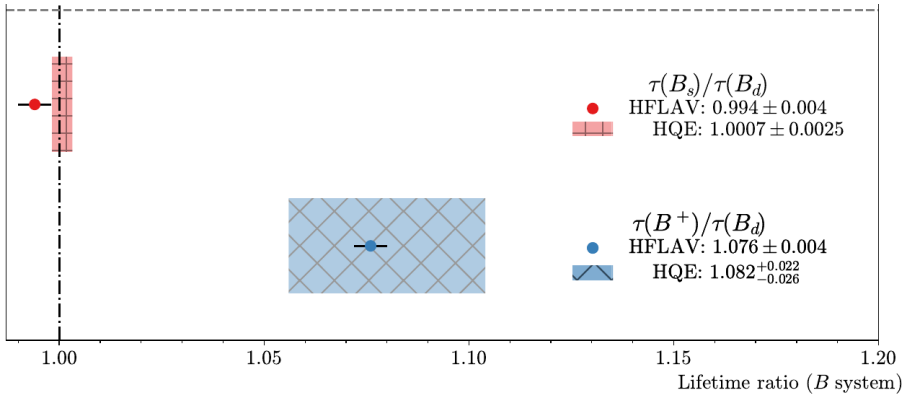


Avg. '19 (2σ excl.) FLAG '19 (2σ excl.) Future '25 (2σ excl.) 2% non-pert./1% V_{cb}

- Assume $\delta C_9^\mu = -\delta C_{10}^\mu$. Simplified Z' and scalar leptoquark model.

Lifetimes, recent progresses

- Dimension-six matrix elements from sum rules
  - Calculation proceeds along the lines of mass-difference calculation



[Kirk,Lenz,Rauh'17]

Lifetimes, recent progresses

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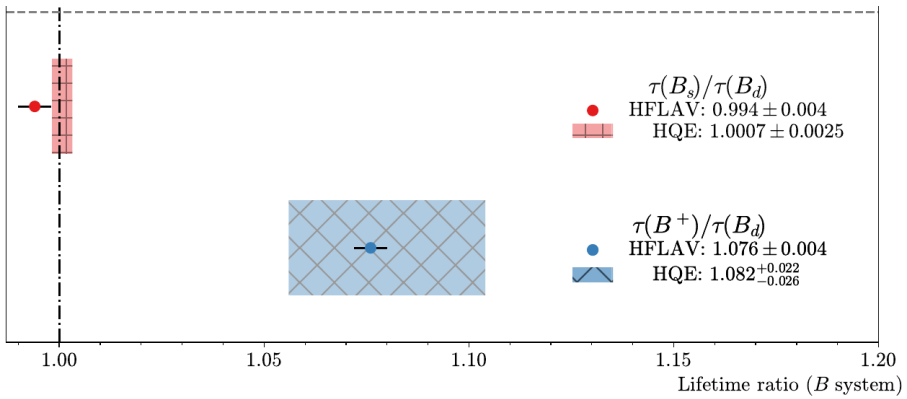
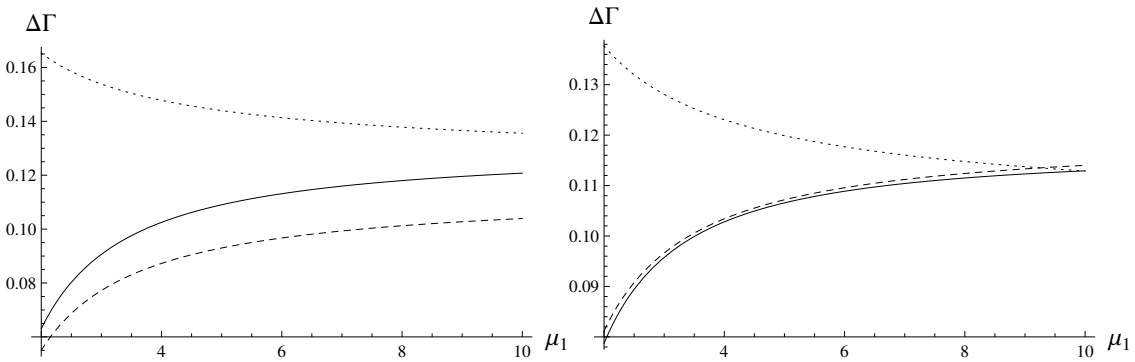
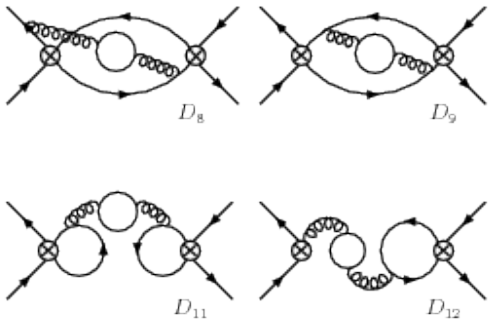


[Kirk,Lenz,Rauh'17]

- Calculation of Γ_{12}^q and a_{fs}^q at order $\mathcal{O}(\alpha_s^2 N_f)$
  - Scale dependence still rather large
  - Significantly reduced scheme dependence





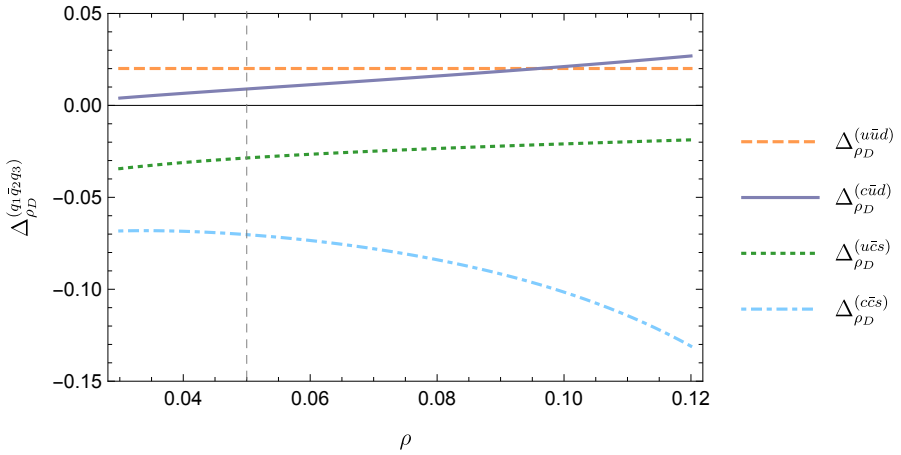
[Nierste et al.'17'20]

Lifetimes

- Dimension-six matrix elements to Darwin ρ_D and Spin-orbit ρ_{LS} term in $\tilde{\Gamma}_3$

  [Mannel,Pivovarov,Moreno'20;Lenz,Piscopo,Rusov'20]

  - Compute matching coefficients C_{ρ_D} and $C_{\rho_{LS}}$



$$\Gamma(b \to c\bar{q}_1 q_2) = \Gamma^0 \left[C_0 - C_{\mu_\pi} \frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G} \frac{\mu_G^2}{2m_b^2} - C_{\rho_D} \frac{\rho_D^3}{2m_b^3} - C_{\rho_{LS}} \frac{\rho_{LS}^3}{2m_b^3} \right]$$

Lifetimes

- Dimension-six matrix elements to Darwin ρ_D and Spin-orbit ρ_{LS} term in $\tilde{\Gamma}_3$

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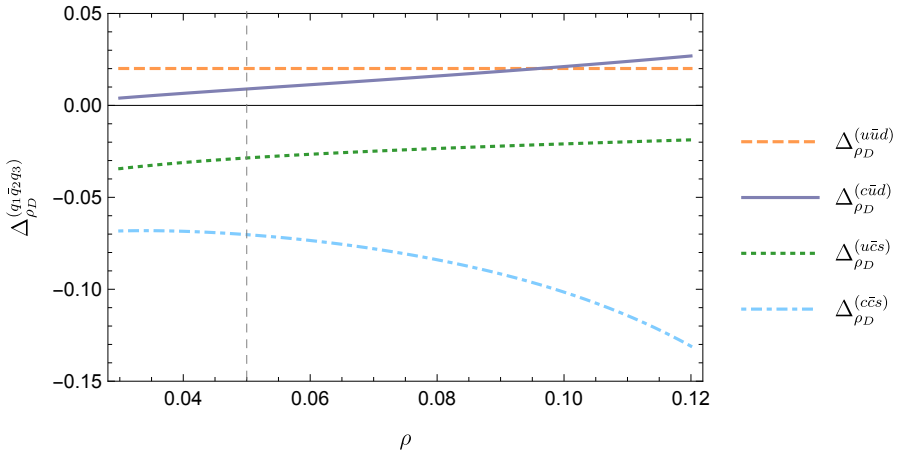
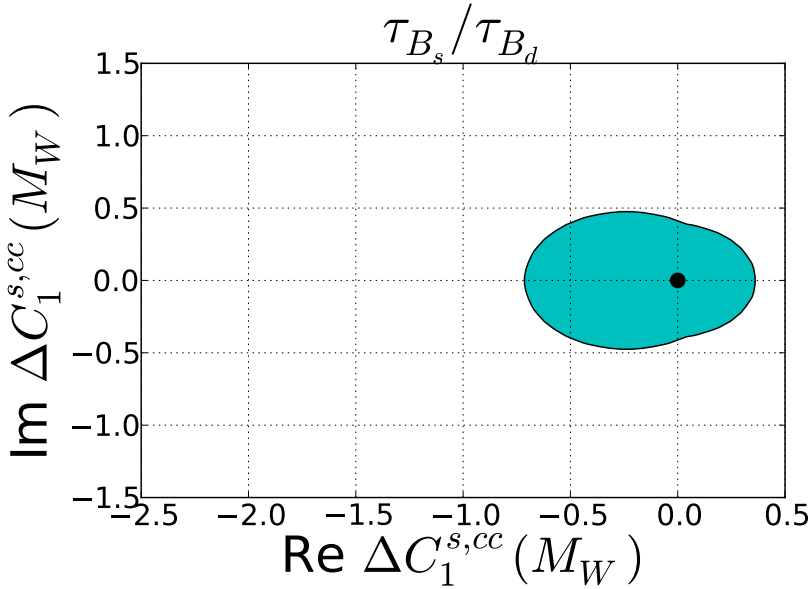
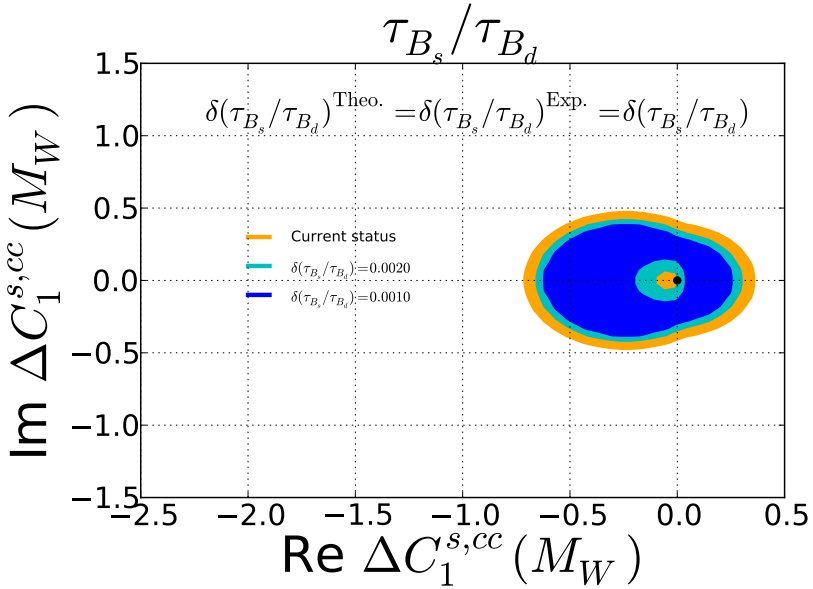


$$\Gamma(b \to c\bar{q}_1 q_2) = \Gamma^0 \left[C_0 - C_{\mu_\pi} \frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G} \frac{\mu_G^2}{2m_b^2} - C_{\rho_D} \frac{\rho_D^3}{2m_b^3} - C_{\rho_{LS}} \frac{\rho_{LS}^3}{2m_b^3} \right]$$

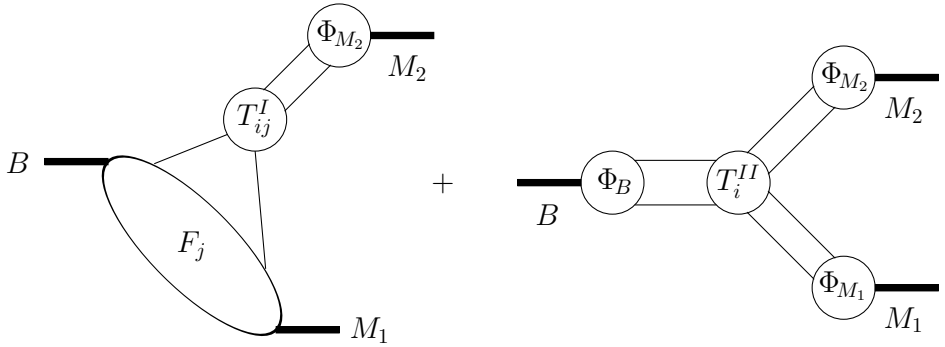
- Combine with non-leptonic tree-level decays

[Tetlalmatzi,Lenz'19]





QCD factorisation for nonleptonic decays



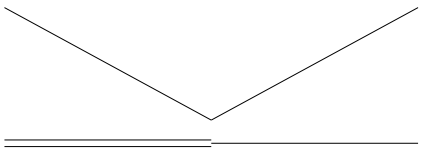
- Amplitude in the limit $m_b \gg \Lambda_{\rm QCD}$ [Beneke,Buchalla,Neubert,Sachrajda'99-'04]

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 \; F_+^{B\to M_1}(0) \; f_{M_2} \int_0^1 du \;\; T_i^I(u) \; \phi_{M_2}(u) \;\; + \;\; (M_1 \leftrightarrow M_2)$$
$$+ f_B \; f_{M_1} \; f_{M_2} \int_0^\infty d\omega \int_0^1 dv \, du \;\; T_i^{II}(\omega, v, u) \; \phi_B(\omega) \; \phi_{M_1}(v) \; \phi_{M_2}(u) \;\; + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

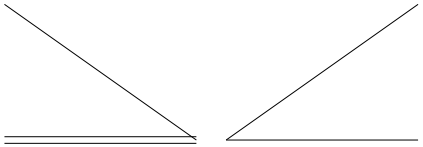
- $T^{I,II}$: Hard scattering kernels, perturbatively calculable
- $F_+ : B \to M$ form factor
  f_i : decay constants
  ϕ_i : light-cone distribution amplitudes
  } Universal. From Sum Rules, Lattice
- Strong phases are $\mathcal{O}(\alpha_s)$ and/or $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$

Classification of amplitudes

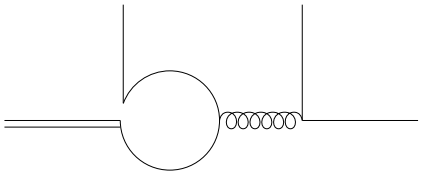
- α_1 : colour-allowed tree amplitude



- α_2 : colour-suppressed tree amplitude



- $\alpha_4^{u,c}$: QCD penguin amplitudes



$$
\begin{aligned}
\sqrt{2}\,\langle\pi^-\pi^0|\,\mathcal{H}_{eff}\,|B^-\rangle &= A_{\pi\pi}\,\lambda_u\Big[\alpha_1(\pi\pi)+\alpha_2(\pi\pi)\Big] \\
\langle\pi^+\pi^-|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle &= A_{\pi\pi}\,\Big\{\lambda_u\Big[\alpha_1(\pi\pi)+\alpha_4^u(\pi\pi)\Big]+\lambda_c\,\alpha_4^c(\pi\pi)\Big\} \\
-\,\langle\pi^0\pi^0|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle &= A_{\pi\pi}\,\Big\{\lambda_u\Big[\alpha_2(\pi\pi)-\alpha_4^u(\pi\pi)\Big]-\lambda_c\,\alpha_4^c(\pi\pi)\Big\}
\end{aligned}
$$

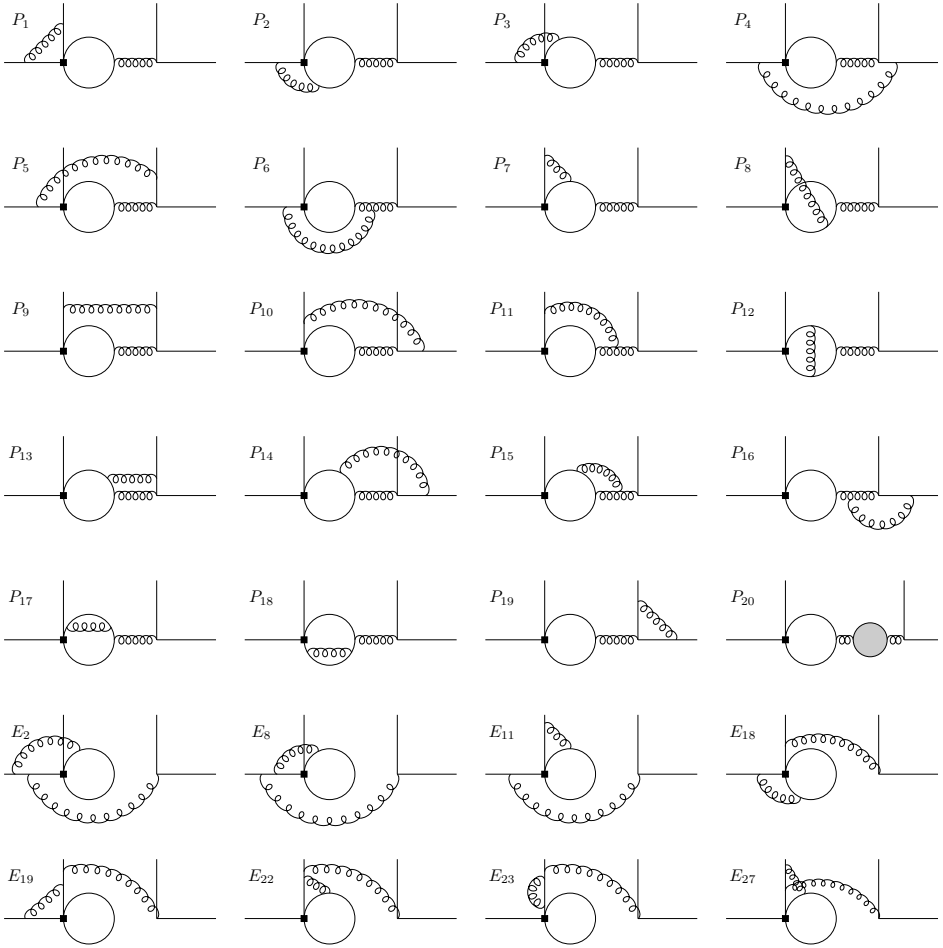
$$
\begin{aligned}
\langle\pi^-\bar{K}^0|\,\mathcal{H}_{eff}\,|B^-\rangle &= A_{\pi\bar{K}}\,\Big[\lambda_u^{(s)}\,\alpha_4^u+\lambda_c^{(s)}\,\alpha_4^c\Big] \\
\langle\pi^+K^-|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle &= A_{\pi\bar{K}}\Big[\lambda_u^{(s)}\,(\alpha_1+\alpha_4^u)+\lambda_c^{(s)}\,\alpha_4^c\Big]
\end{aligned}
$$

[Beneke,Neubert'03]

- Tree amplitudes α_1 and α_2 known analytically to NNLO [Bell'07'09; Beneke,Li,TH'09]

Penguin amplitudes at two loops

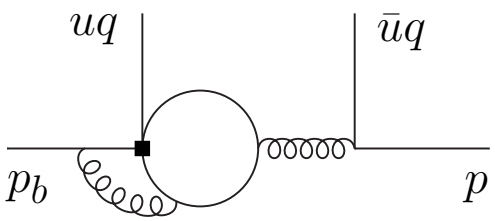
- Contributing two-loop penguin diagrams



- Plus: Q_{3-6} insertion into tree topology (66 diagrams, known)
  and one-loop diagrams involving Q_8 (18 diagrams, simple)

Kinematics

- Kinematics:



$$p^2 = q^2 = 0$$
$$p_b^2 = m_b^2$$

  - Fermion loop with either $m = 0$ or $m = m_c$.
  - Genuine two-scale problem: $\bar{u}$, m_c^2/m_b^2
  - Threshold at $\bar{u} = 4m_c^2/m_b^2$
  - Choice of suitable kinematic variables crucial

$$s = \sqrt{1 - 4z_c/\bar{u}}\ ,\ r = \sqrt{1 - 4z_c} \quad \longleftrightarrow \quad \bar{u}\ ,\ z_c = \frac{m_c^2}{m_b^2} \quad \longleftrightarrow \quad s_1 = \sqrt{1 - 4/\bar{u}}\ ,\ r$$

$$\updownarrow$$

$$p = \frac{1 - \sqrt{u^2 + 4\bar{u}z_c}}{\bar{u}}\ ,\ r$$

Two-loop calculation

- Regularize UV and IR divergences dimensionally. Poles up to $1/\epsilon^3$
- Reduction: IBP relations, Laporta algorithm in FIRE, get 36 master integrals

  [Tkachov'81; Chetyrkin,Tkachov'81] [Laporta'01; Smirnov'08]
- Use differential equations in canonical form [Henn'13]

$$d\,\vec{M}(\epsilon, x_n) \;=\; \epsilon\;\; d\tilde{A}(x_n)\;\; \vec{M}(\epsilon, x_n)$$

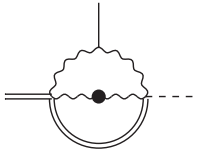
  - Analytic solution in terms of iterated integrals (GPLs) over alphabet [Bell,TH'14]

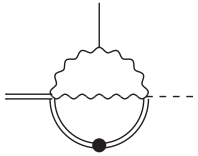
$$\left\{0\,,\;\pm 1\,,\;\pm 3\,,\;\pm i\sqrt{3}\,,\;\pm r\,,\;\pm\frac{r^2+1}{2}\,,\;\pm(1+2\sqrt{z_c})\,,\;\pm(1-2\sqrt{z_c})\right\}$$

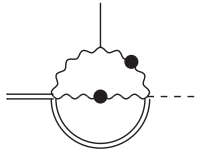
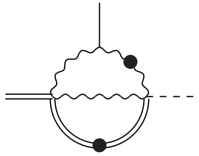
  - Catalyses analytic convolution with LCDA
- UV renormalisation, IR subtraction, matching onto SCET, pole cancellation
- Convolution with LCDA $\displaystyle\int_0^1 du\;\; T_i(u)\;\phi_M(u)$
  - Expand LCDA of light meson in Gegenbauer polynomials

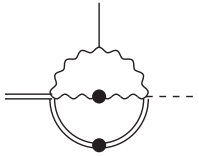
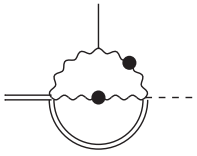
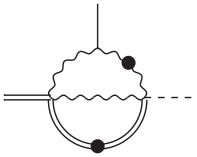
$$\phi_M(u) = 6u\bar{u}\left[1+\sum_{n=1}^{\infty} a_n^M\, C_n^{(3/2)}(2u-1)\right]$$

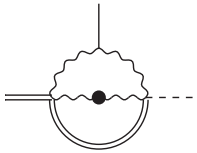
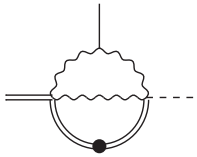
Canonical basis for master integrals

$\frac{M_{18}}{u\epsilon^3} =$ 

$\frac{M_{19}}{u\epsilon^3} =$ 

$-\frac{2\,M_{20}}{u\bar{u}s\epsilon^2} =$  $+$ 

$\frac{M_{21}}{\epsilon^2} = \frac{2[(1+\bar{u})^2 z_c - \bar{u}^2]}{\bar{u}}$  $- \bar{u}s^2(1+\bar{u})\Big[$  $+$  $\Big]$

$+\frac{2\epsilon u}{m_b^2}\Big[$  $+$  $\Big]$

- Differential equation (sample)

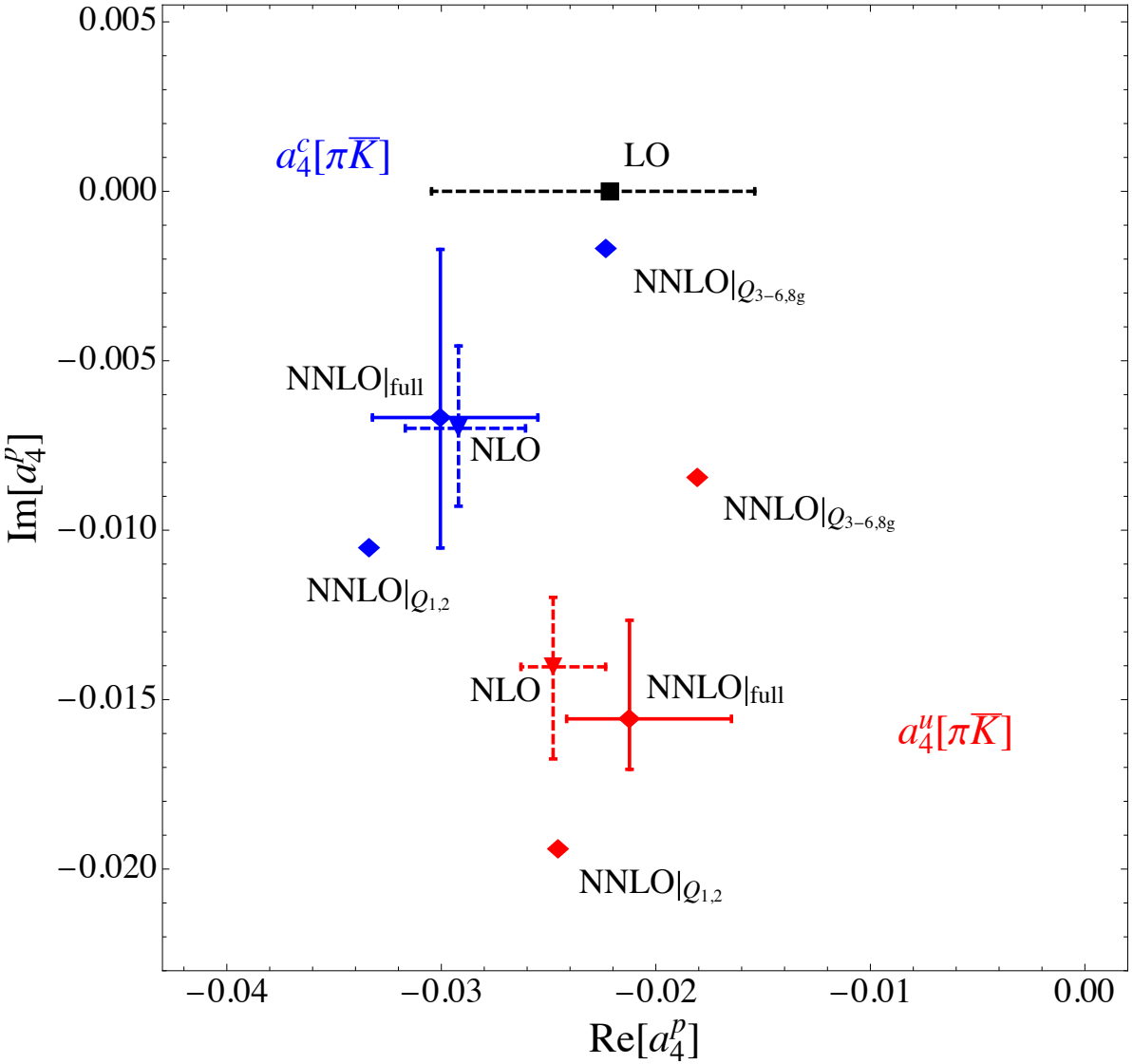
$$\frac{dM_{19}}{ds} = \frac{4\epsilon M_{18}\, r\, \left(r^2+1\right)}{((r^2+1)^2-4s^2)} - \frac{2\epsilon M_{19}\, r\, \left(r^2+s^2-2\right)}{(1-r^2)(r^2-s^2)} + \frac{4\epsilon M_{20}\, r\, s}{\left((r^2+1)^2-4s^2\right)} - \frac{\epsilon M_{21}\, r\, \left(r^2+1\right)}{((r^2+1)^2-4s^2)}$$

- Boundary conditions
  - M_{18} and M_{19} vanish in $s = r$ (i.e. in $u = 0$)
  - M_{20} and M_{21} vanish in $s = +i\infty$ (i.e. in $u = 1$)

Results

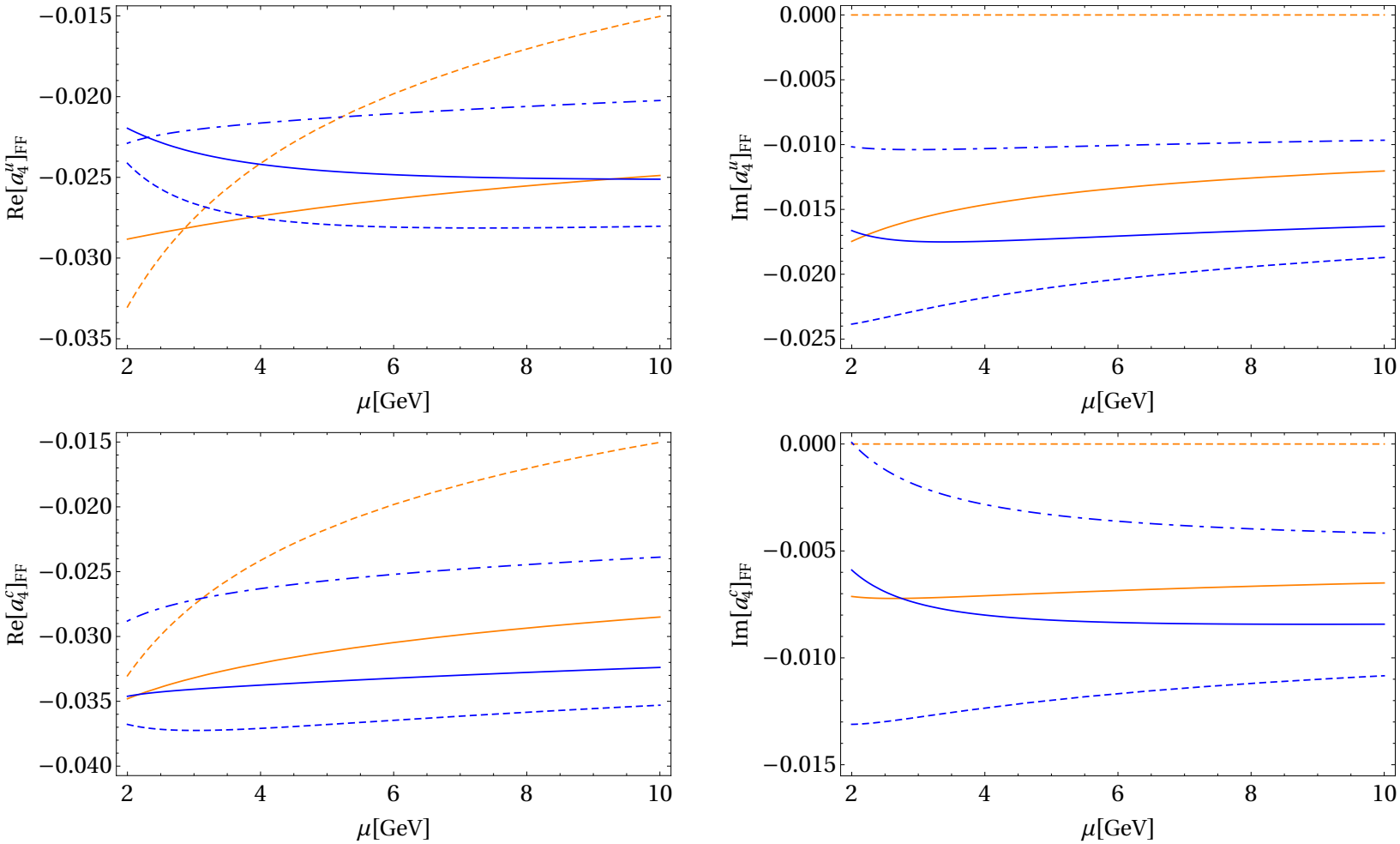
- Penguin amplitudes: Anatomy of corrections

[Bell,Beneke,Li,TH'20]



Results

- Penguin amplitudes: Scale dependence. [Bell,Beneke,Li,TH'20]
  - Only form factor term, no spectator scattering.
- a_4^u
- a_4^c



- Orange: LO (dashed) and NLO (solid) Blue: NNLO $Q_{1,2}$ (d), $Q_{3-6,8g}$ (dd), all (s)

Results: Direct CP asymmetries

[Bell,Beneke,Li,TH'15]

- Direct CP asymmetries in percent.

  Errors are CKM and hadronic, respectively.

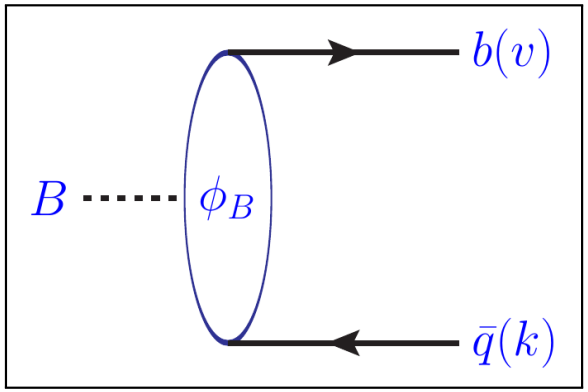
f	NLO	NNLO	NNLO + LD	Exp
$\pi^-\bar{K}^0$	$0.71^{+0.13\,+0.21}_{-0.14\,-0.19}$	$0.77^{+0.14\,+0.23}_{-0.15\,-0.22}$	$0.10^{+0.02\,+1.24}_{-0.02\,-0.27}$	-1.7 ± 1.6
$\pi^0 K^-$	$9.42^{+1.77\,+1.87}_{-1.76\,-1.88}$	$10.18^{+1.91\,+2.03}_{-1.90\,-2.62}$	$-1.17^{+0.22\,+20.00}_{-0.22\,-\,\,6.62}$	4.0 ± 2.1
$\pi^+ K^-$	$7.25^{+1.36\,+2.13}_{-1.36\,-2.58}$	$8.08^{+1.52\,+2.52}_{-1.51\,-2.65}$	$-3.23^{+0.61\,+19.17}_{-0.61\,-\,\,3.36}$	-8.2 ± 0.6
$\pi^0\bar{K}^0$	$-4.27^{+0.83\,+1.48}_{-0.77\,-2.23}$	$-4.33^{+0.84\,+3.29}_{-0.78\,-2.32}$	$-1.41^{+0.27\,+5.54}_{-0.25\,-6.10}$	1 ± 10
$\delta(\pi\bar{K})$	$2.17^{+0.40\,+1.39}_{-0.40\,-0.74}$	$2.10^{+0.39\,+1.40}_{-0.39\,-2.86}$	$2.07^{+0.39\,+2.76}_{-0.39\,-4.55}$	12.2 ± 2.2
$\Delta(\pi\bar{K})$	$-1.15^{+0.21\,+0.55}_{-0.22\,-0.84}$	$-0.88^{+0.16\,+1.31}_{-0.17\,-0.91}$	$-0.48^{+0.09\,+1.09}_{-0.09\,-1.15}$	-14 ± 11

$$\delta(\pi\bar{K}) = A_{\rm CP}(\pi^0 K^-) - A_{\rm CP}(\pi^+ K^-)$$

$$\Delta(\pi\bar{K}) = A_{\rm CP}(\pi^+ K^-) + \frac{\Gamma_{\pi^-\bar{K}^0}}{\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^-\bar{K}^0) - \frac{2\Gamma_{\pi^0 K^-}}{\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^0 K^-) - \frac{2\Gamma_{\pi^0\bar{K}^0}}{\Gamma_{\pi^+K^-}} A_{\rm CP}(\pi^0\bar{K}^0)$$

LCDA of the B-meson

Parton picture: 2-particle Fock state



- Some external light-like momentum, e.g. $p_\gamma^\mu = E_\gamma n^\mu\,,\ n^2 = 0$
- with $\omega \equiv n \cdot k$ light-cone projection of light antiquark momentum
- $\phi_B(\omega)$ as probability amplitude

- Field theoretical definition of $\phi_B^+(\omega)$ from light-cone operators in HQET:

$$m_B f_B^{(\mathrm{HQET})}\ \phi_B^+(\omega) = \int \frac{d\tau}{2\pi}\, e^{i\omega\tau}\, \langle 0|\, \bar{q}(\tau n)\ [\tau n,\, 0]\, \not{n}\gamma_5\ h_v^{(b)}(0)\, |\bar{B}(m_B v)\rangle$$

[Grozin,Neubert'96]

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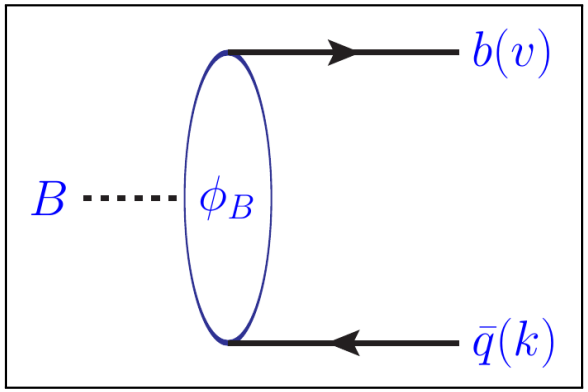

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- in QCD factorization theorems one encounters **logarithmic moments**

$$L_n(\mu) \quad = \quad \int_0^\infty \frac{d\omega}{\omega}\, \ln^n\left(\frac{\omega}{\mu}\right) \phi_B^+(\omega) \qquad \text{(n=0,1,2, ...)}$$

- in QCD light-cone sum rules, one is sensitive to **low light-cone momenta**

$$\phi_B^{+\prime}(0)\,, \qquad \phi_B^{+\prime\prime}(0)\,, \qquad \text{etc.}$$

Theoretical developments

- 1-loop RGE [Lange,Neubert'03]
- Radiative tail including dim-5 HQET operators [Kawamura,Tanaka'08]

$$\tilde{\phi}_+(\tau) \overset{\tau\to 0}{=} 1 - \frac{\alpha_s C_F}{4\pi}\left(2\ln^2(i\tau\mu) + 2\ln(i\tau\mu) + \frac{5\pi^2}{12}\right) + \mathcal{O}(\tau) + \mathcal{O}(\tau^2)$$

- Eigenfunctions of 1-loop RGE [Bell,Feldmann,Wang'13;Braun,Manashov'14]

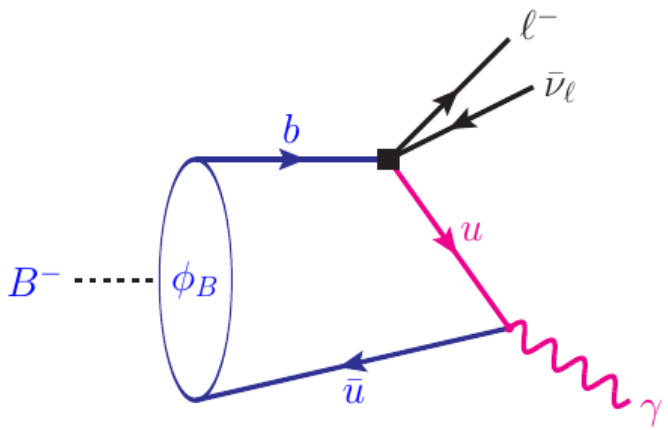
$$\phi_B^+(\omega;\mu) = \int_0^\infty ds\,\sqrt{\omega s}\,J_1(2\sqrt{\omega s})\,U(s;\mu,\mu_0)\,\eta_B^+(s;\mu_0)$$

- 2-loop RGE from conformal symmetry [Braun,Ji,Manashov'19]

$$\left(\frac{d}{d\ln\mu} + \Gamma_c\,\ln(\mu\,s\,e^{2\gamma_E}) + \gamma_+\right)\eta_B^+(s,\mu) = 4C_F\left(\frac{\alpha_s}{4\pi}\right)^2\int_0^1 du\,\frac{\bar{u}}{u}\,h(u)\,\eta_B^+(\bar{u}s,\mu)$$

- 2-loop evolution for generating function of log. moments [Galda,Neubert'20]

Simplest application: $B \to \gamma$ form factors



Consider $\bar{B} \to \gamma\, \ell\nu$

For large photon energy, $E_\gamma \sim m_b/2$:

$$(p_\gamma - p_{\bar{u}})^2 \simeq -2\, p_\gamma \cdot p_{\bar{u}} \equiv -2\, E_\gamma\, \omega$$

- Sensitive to light-cone projection ω of light antiquark momentum in B-meson

$$F^{B\to\gamma}(E_\gamma) \;\simeq\; \text{[kinematic factor]} \times \int_0^\infty \frac{d\omega}{\omega}\, \phi_B(\omega)$$

Experimental bound on BR $\longrightarrow$ bound on $\dfrac{1}{\lambda_B} \equiv \displaystyle\int_0^\infty \frac{d\omega}{\omega}\, \phi_B(\omega)$

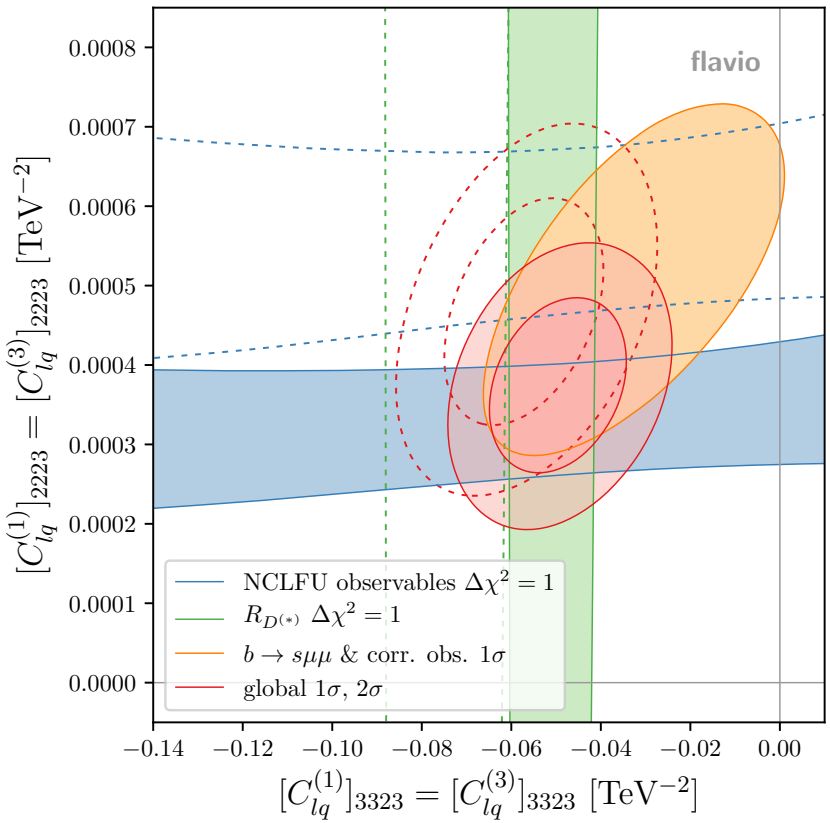
[Beneke,Rohrwild'11;Braun,Khodjamirian'12;Beneke,Braun,Ji,Wei'18]

SMEFT

- Standard Model effective field theory (SMEFT)
  - Basis of dim.-6 operators made of SM fields and with SM gauge symmetry

[Buchmüller,Wyler'86;Grzadkowski,Iskrzynski,Misiak,Rosiek'10;Alonso,Jenkins,Manohar,Trott'13;see also Buchalla,Cata,Krause'13; Pomarol et al.'13]

  - Approx. 60 operators, proliferates to ~ 2500 with flavour indices
- Look at various subsets of Wilson coefficients, find constraints with global fits



- Scenario: only four non-zero SMEFT Wilson coefficients at 2 TeV
- Solid (dashed) contours include (exclude) Moriond-2019 results for R_K, R_{K^*}, R_D, and R_{D^*}

[Aebischer,Altmannshofer,Guadagnoli,Reboud,Stangl,Straub'19]

Conclusion and Outlook

- Many of the yet unanswered questions of particle physics are related to the Yukawa sector of the SM
- Flavour sector of the SM is currently being investigated to unprecedented precision
- Description of quark flavour sector benefits from interplay of many different aspects, many of which are also important in other branches of particle physics

Effective theories	QCD & QED corrections	Lattice, Sum rules
Factorisation	Experiment	BSM

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Effective theories	QCD & QED corrections	Lattice, Sum rules
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- Exciting times in particle physics are ahead of us
- Flavour physics will help to (hopefully) reveal and quantify the remaining mysteries in particle physics