Precision flavour physics

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Outline

- Introduction / motivation
- Charged-current semileptonic decays
- Radiative decays
- FCNC semileptonic decays
- Mixing and lifetimes
- Nonleptonic decays
- SMEFT
- Conclusion and outlook
Introduction

- The majority of the SM parameters resides in the Yukawa sector
  - Quarks and leptons are the principal actors of flavour physics
- Many aspects of flavour physics
  - Heavy (top, bottom, charm) and light quarks
  - Mesons and baryons
  - Charged leptons, neutrinos
Introduction

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- Quarks and leptons are the principal actors of flavour physics

Many aspects of flavour physics

- Heavy (top, bottom, charm) and light quarks
- Mesons and baryons
- Charged leptons, neutrinos

Have to make a selection:
Will focus on $B$ mesons, because . . .

Description of $B$-meson system contains many essential features of (precision) flavour physics

- Global fits driven by $B$ meson anomalies
Motivation to study flavour

- CP violation
  - Needed to explain matter-antimatter asymmetry (Sakharov conditions)
  - CKM phase is the only established source of CP violation in the SM
  - But too small to explain size of baryon asymmetry
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- Indirect search for new physics
  - Look for virtual effects of new phenomena
  - Mostly looked for in rare processes
  - Requires precision in theory and experiment
  - Synergy and complementarity to direct searches

Huge experimental progress (B-factories, Tevatron, LHC, Belle II, ...)

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Motivation to study flavour

- **SM parameters**
  - Precise knowledge of masses and mixing parameters needed in all branches of particle physics
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Motivation to study flavour

- SM parameters
  - Precise knowledge of masses and mixing parameters needed in all branches of particle physics

- Problem: confinement of quarks into hadrons
  - Computation of hadronic matrix elements highly non-trivial
  - QCD effects could overshadow the interesting fundamental dynamics

- Need to get control over QCD effects
  - Sophisticated tools available
    - Effective field theories (HQET, SCET, ...)
    - Heavy-Quark expansion
    - Factorization
    - Perturbative calculations: Loops, ... 
    - Non-pert. techniques: Lattice, Sum rules, ... 
    - Applications also in Higgs, Collider, DM, ...
Effective theory for $B$ decays

- $M_W, M_Z, m_t, m_H \gg m_b$: integrate out heavy gauge bosons, $t$-quark, Higgs

- Effective Weak Hamiltonian:

\[ H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left[ C_1 Q_1^p + C_2 Q_2^p + \sum_{k=3}^{10} C_k Q_k \right] + \text{h.c.} \]

\[ Q_1^p = (\bar{d}_L \gamma^\mu T^a p_L)(\bar{p}_L \gamma_\mu T^a b_L) \]
\[ Q_2^p = (\bar{d}_L \gamma_\mu p_L)(\bar{p}_L \gamma_\mu b_L) \]
\[ Q_3 = (\bar{d}_L \gamma^\mu b_L) \sum_q (\bar{q} \gamma_\mu q) \]
\[ Q_4 = (\bar{d}_L \gamma^\mu T^a b_L) \sum_q (\bar{q} \gamma_\mu T^a q) \]
\[ Q_5 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q) \]
\[ Q_6 = (\bar{d}_L \gamma^\mu \gamma^\nu \gamma^\rho T^a b_L) \sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^a q) \]
\[ Q_7 = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma_{\mu\nu} F_{\mu\nu} b_R \]
\[ Q_8 = \frac{g_s}{16\pi^2} m_b \bar{s}_L \sigma_{\mu\nu} G_{\mu\nu} b_R \]
\[ Q_9 = (\bar{s}_L \gamma^\mu b_L)(\bar{\ell} \gamma_\mu \ell) \]
\[ Q_10 = (\bar{s}_L \gamma^\mu b_L)(\bar{\ell} \gamma_\mu \gamma_5 \ell) \]
\[ \lambda_p = V_{pb} V_{pd}^* \]
Effective theory for $B$ decays

- Generic structure of amplitude for $B$ decays

$$\mathcal{A}(\bar{B} \rightarrow f) = \sum_i \left[ \lambda_{\text{CKM}} \times C \times \langle f|\mathcal{O}|\bar{B}\rangle_{\text{QCD+QED}} \right]_i$$

- Interplay between

  - Wilson coefficients $C$, known to NNLL in SM, values at scale $\mu \sim m_b$

    [Bobeth,Misiak,Urban'99;Misiak,Steinhauser'04,Gorbahn,Haisch'04;Gorbahn,Haisch,Misiak'05;Czakon,Haisch,Misiak'06]

    $$C_1 = -0.25 \quad |C_{3,5,6}| < 0.01 \quad C_7 = -0.30 \quad C_9 = 4.06$$

    $$C_2 = 1.01 \quad C_4 = -0.08 \quad C_8 = -0.15 \quad C_{10} = -4.29$$

  - CKM factors $\lambda_{\text{CKM}}$. Hierarchy of CKM elements, weak phase

  - Hadronic matrix elements $\langle f|\mathcal{O}|\bar{B}\rangle$. Can contain strong phases.

- Interplay offers rich and interesting phenomenology for $B$ decays

  - Plethora of data, numerous observables

  - Test of CKM mechanism and indirect search for New Physics

- BUT: Challenging QCD dynamics in hadronic matrix elements.

  Effects from many different scales !!
Inclusive $B$ decays, generalities

- Main tool for inclusive decays: Heavy Quark Expansion

    \[ \Gamma(B_q \to X) = \frac{1}{2m_{B_q}} \sum_X \int_{PS} (2\pi)^4 \delta^{(4)}(p_{B_q} - p_X) |\langle X | \hat{H}_{eff} | B_q \rangle|^2 \]

- Use optical theorem

    \[ \Gamma(B_q \to X) = \frac{1}{2m_{B_q}} \langle B_q | \hat{T} | B_q \rangle \quad \text{with} \quad \hat{T} = \text{Im} i \int d^4x \hat{T} \left[ \hat{H}_{eff}(x) \hat{H}_{eff}(0) \right] \]

- Expand non-local double insertion of effective Hamiltonian in local operators

    \[ \Gamma = \Gamma_0 \langle O_{D=3} \rangle + \Gamma_2 \frac{\langle O_{D=5} \rangle}{m_b^2} + \tilde{\Gamma}_3 \frac{\langle \tilde{O}_{D=6} \rangle}{m_b^3} + \ldots \]

    \[ + 16\pi^2 \left[ \Gamma_3 \frac{\langle O_{D=6} \rangle}{m_b^3} + \Gamma_4 \frac{\langle O_{D=7} \rangle}{m_b^4} + \Gamma_5 \frac{\langle O_{D=8} \rangle}{m_b^5} + \ldots \right] \]

- Expand each term in perturbative series

    \[ \Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_i^{(1)} + \left( \frac{\alpha_s}{4\pi} \right)^2 \Gamma_i^{(2)} + \ldots . \]
Gamma 0: Decay of a free quark, known to $O(\alpha_s^2)$
Gamma 1: Vanishes due to Heavy Quark Symmetry

Two terms in $\Gamma_2$

- Kinetic energy $\mu_{\pi}$:
  \[2M_B \mu_{\pi}^2 = -\langle B(v) | \bar{b}_v (iD)^2 b_v | B(v) \rangle\]

- Chromomagnetic moment $\mu_G$:
  \[2M_B \mu_G^2 = -i \langle B(v) | \bar{b}_v \sigma_{\mu\nu} (iD^\mu)(iD^\nu) b_v | B(v) \rangle\]

  Both known to $O(\alpha_s)$ [Becher, Boos, Lunghi’07; Alberti, Ewerth, Gambino, Nandi’13’14; Mannel, Pivovarov, Rosenthal’15]

Two more terms in $\Gamma_3$

- Darwin term $\rho_D$:
  \[2M_H \rho_D^3 = -\langle B(v) | \bar{b}_v (iD_\mu)(ivD)(iD^\mu) b_v | B(v) \rangle\]

- Spin-orbit term $\rho_{LS}$:
  \[2M_H \rho_{LS}^3 = -i \langle B(v) | \bar{b}_v \sigma_{\mu\nu} (iD^\mu)(ivD)(iD^\nu) b_v | B(v) \rangle\]

  Recently, $\rho_D$ became available at $O(\alpha_s)$ [Mannel, Pivovarov’19]
Higher orders in HQE and inclusive $B \rightarrow X_c \ell \bar{\nu}_\ell$

- $\rho_D$ at $\mathcal{O}(\alpha_s)$

$$\Gamma(B \rightarrow X_c \ell \bar{\nu}_\ell) = \Gamma_0 |V_{cb}|^2 \left[ a_0 \left( 1 + \frac{\mu^2}{2m_b^2} \right) + a_2 \frac{\mu_G^2}{2m_b^2} + \frac{a_D \rho_D + a_{LS} \rho_{LS}}{2m_b^3} + \ldots \right]$$

- One finds $a_D = -57.159 \left( 1 - \frac{\alpha_s}{4\pi} 6.564 \right) = -57.159 \left( 1 - 0.10 \right)$

- Number of parameters grows factorially at higher orders in $1/m$

- Partial reduction by reparametrisation invariance

[Mannel,Pivovarov'19]

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10]
Exclusive $B$ decays, generalities

- Leptonic decays

\[ \langle 0|\bar{u}\gamma^\mu\gamma_5 b|B^- (p)\rangle = i f_B p^\mu \]
Exclusive $B$ decays, generalities

- **Leptonic decays**

\[
\langle 0|\bar{u}\gamma^\mu\gamma_5 b|B^-(p)\rangle = i f_B \, p^\mu
\]

- **Semi-leptonic decays**

\[
\langle D(p')|\bar{c}\gamma^\mu b|\bar{B}(p)\rangle = F_+(q^2) \left[ (p + p')^\mu - \frac{m_B^2 - m_D^2}{q^2} q^\mu \right] + F_0(q^2) \frac{m_B^2 - m_D^2}{q^2} q^\mu
\]
Exclusive $B$ decays, generalities

- Leptonic decays

$\langle 0 | \bar{u} \gamma^\mu \gamma_5 b | B^- (p) \rangle = i f_B \, p^\mu$

- Semi-leptonic decays

$\langle D(p') | \bar{c} \gamma^\mu b | \bar{B}(p) \rangle$

$= F_+(q^2) \left[ (p + p')^\mu - \frac{m_B^2 - m_D^2}{q^2} q^\mu \right]$ $+ F_0(q^2) \frac{m_B^2 - m_D^2}{q^2} q^\mu$

- Non-leptonic decays

$\langle \pi^- D^+ | Q_i | \bar{B} \rangle \simeq m_B^2 \, f_{M_2} \, F_+^{B \rightarrow D}(m_\pi^2)$ $\times \int_0^1 du \, T_i^I(u) \, \phi_\pi(u)$
Exclusive $B \rightarrow D^{(*)}\ell\bar{\nu}_\ell$ decays

- Decay rates in terms of form factors \( (w = v_B \cdot v_{D^{(*)}}) \)

\[
\frac{d\Gamma(\bar{B} \rightarrow D^* \ell \bar{\nu})}{dw} = \frac{G_F^2|V_{cb}|^2}{48\pi^3} \frac{(m_B - m_{D^*})^2 m_{D^*}^3}{m_B^2 - 2w m_B m_{D^*} + m_{D^*}^2} \sqrt{w^2 - 1} (w + 1)^2 m_B^2 \sqrt{w^2 - 1} \left(1 + 4w\right) \frac{m_B^2 - 2w m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2} |F(w)|^2
\]

\[
\frac{d\Gamma(\bar{B} \rightarrow D \ell \bar{\nu})}{dw} = \frac{G_F^2|V_{cb}|^2}{48\pi^3} \frac{(m_B + m_D)^2 m_D^3 (w^2 - 1)^{3/2}}{m_B^2 - 2w m_B m_D + m_D^2} \left|V_1(w)\right|^2
\]

- Form factor parametrisations

  - CLN: In terms of an intercept and a slope [Caprini,Lellouch,Neubert'97]

  - BGL: More general, based on dispersion relations, analyticity, and crossing symmetry [Boyd,Grinstein,Lebed'94’95’97]
Recent result from global fit

- Difference is currently at the $1.9\sigma$ level

Inclusive vs. exclusive $|V_{cb}|$

- $B\to X_c$  BarBar/Belle'04-'10, [3]
- $B\to D$  BaBar'09+Belle'16, [4–6]
- $B\to D^*$  Belle'17, [2,13,18]
- $B\to D^*$  Belle'18, [2,18] + this work
- $B\to D^*$  Belle'17'18, [2,18] + this work
Inclusive $\bar{B} \rightarrow X_s \gamma$

- One of the standard candles in the search for NP in the quark flavour sector
- Flavour-changing neutral current process
- Dominant contribution is loop-induced
  - Indirectly sensitive to new particles
- Plays a prominent role in global fits

Current SM prediction vs. measurement (for $E_\gamma > 1.6$ GeV)

$$B_{s\gamma}^{\text{SM}} = (3.40 \pm 0.17) \times 10^{-4} \quad [\text{Misiak, Rehman, Steinhauser'20}]$$

$$B_{s\gamma}^{\text{exp.}} = (3.32 \pm 0.15) \times 10^{-4} \quad [\text{HFLAV'19}]$$

SM prediction is based on the formula

$$B(\bar{B} \rightarrow X_s \gamma)_{E_\gamma > E_0} = B(\bar{B} \rightarrow X_c \ell \bar{\nu}) \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6 \alpha_{em}}{\pi} \left[ P(E_0) + N(E_0) \right] C,$$
Perturbative corrections

- Perturbative corrections to partonic decay width far advanced
- Many corrections through to NNLO known
- One of the largest uncertainties ($\sim 3\%$) comes from interpolation in $m_c$ in $Q_{1,2} - Q_7$ interference

Currently underway

- Exact charm-mass dependence of $Q_{1,2} - Q_7$ interference at NNLO
- One-loop multi-body contributions, formally NLO but suppressed

Large $m_c$ expansion
Calculation f. $m_c = 0$
Exact $m_c$ dependence of fermionic part
Exact $m_c$ dependence of full result
Perturbative corrections

- One-loop multi-body contributions to partonic process $b \rightarrow s q\bar{q}\gamma$
- Formally NLO but suppressed by CKM factors or small Wilson coefficients

One-loop four-particle cuts
- Tree-level five-particle cuts
- IBP reduction (reversed unitarity)
- Phase space integrations
- See Lars Moos’ talk at YSF
Nonperturbative quantities

Semileptonic phase space factor

\[ C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma[\bar{B} \to X_c e \bar{\nu}]}{\Gamma[\bar{B} \to X_u e \bar{\nu}]} \]

\[ = g(z) \left\{ 0.903 - 0.588 [\alpha_s(4.6 \text{ GeV}) - 0.22] + 0.0650 [m_{b,\text{kin}} - 4.55] \right\} \]

\[ - 0.1080 [m_c(2 \text{ GeV}) - 1.05] - 0.0122 \mu_G^2 - 0.199 \rho_D^3 + 0.004 \rho_{LS}^3 \}

Determined by using HQET methods from measurements of \( \bar{B} \to X_c \ell \bar{\nu} \)

[Alberi,Gambino,Healey,Nandi'14]
Nonperturbative quantities

- **Semileptonic phase space factor**

\[ C = \frac{|V_{ub}|^2 \Gamma[\bar{B} \to X_c e \bar{\nu}] \Gamma[\bar{B} \to X_u e \bar{\nu}]}{\Gamma[\bar{B} \to X_c e \bar{\nu}]} \]

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- **Determined by using HQET methods from measurements of \( \bar{B} \to X_c \ell \bar{\nu} \)**

- **Gluon-photon conversion in \( Q_7 - Q_8 \) interference**

\[ \Gamma[B^- \to X_s \gamma] \simeq A + B Q_u + C Q_d + D Q_s, \]
\[ \Gamma[\bar{B}^0 \to X_s \gamma] \simeq A + B Q_d + C Q_u + D Q_s \]
\[ \frac{\delta \Gamma_c}{\Gamma} \simeq \frac{Q_u + Q_d}{Q_d - Q_u} \left[ 1 + 2 \frac{D - C}{C - B} \right] \Delta_0^- = -\frac{1}{3} (1 \pm 0.3) \Delta_0^- = (0.16 \pm 0.74)\% \]

- \( \Delta_0^- \): Isospin asymmetry, measured at Belle
- \( C - D \) vanishes in isospin limit, assume 30% SU(3) breaking
Nonperturbative quantities

- Resolved photon contribution in $Q_{1,2} - Q_7$ interference

  [Voloshin'96; Buchalla, Isidori, Rey'97; Benzke, Hurth, Fickinger, Turczyk'17'20; Gunawardana, Paz'19]

  $$ N(E_0) \sim C_7 \left( C_2 - \frac{1}{6} C_1 \right) \left( - \frac{\mu^2}{27m_c^2} + \frac{\Lambda_{17}}{m_b} \right) \equiv \delta N_V \kappa_V $$

  $$ \Lambda_{17} = e_c \text{Re} \int_{-\infty}^{\infty} \frac{d\omega_1}{\omega_1} \left[ 1 - F \left( \frac{m_c^2 - i\varepsilon}{m_b \omega_1} \right) + \frac{m_b \omega_1}{12m_c^2} \right] h_{17}(\omega_1, \mu) $$

  - $h_{17}$: soft function, modeled in [Gunawardana, Paz'19]

  - Yields $\Lambda_{17} \in [-24, 5]$ MeV $\implies \kappa_V = 1.2 \pm 0.3$

  [see also Benzke, Lee, Neubert, Paz'10; Benzke, Hurth'20]
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Gluon-photon conversion in $Q_7 - Q_8$ interference

- $P(E_0)$ receives corrections $\propto |C_8|^2 \log(\frac{m_b}{m_s})$
- Small contribution (<1%), but large uncertainty
- Vary $\log(\frac{m_b}{m_s}) \in [\log(10), \log(50)]$ [Czakon et al.’15]
- Additional nonperturbative effects $\propto |C_8|^2$
  impact $B_{s\gamma}$ in range $[-0.3, 1.9]$% [Benzke, Lee, Neubert, Paz’10]
- Reproduce numerically by
  \[ \log(\frac{m_b}{m_s}) \to \kappa_{88} \log(50) \]  and \[ \kappa_{88} = 1.7 \pm 1.1 \]
NP constraints

- Charged Higgs mass in two-Higgs doublet models

![Graph showing BR(\bar{B} \rightarrow X_s \gamma) vs. \pm H (GeV) with latest bound \( M_{H^+} > 800 \) GeV at 95% C.L.]

- Updates in 2017 and 2020. Latest bound: \( M_{H^+} > 800 \) GeV at 95% C.L.

[Misiak, Rehman, Steinhauser'20]
Angular analysis of $\bar{B} \to K^* (\to K\pi) \ell^+ \ell^-$

$$
\frac{d^4\Gamma}{dq^2 \, d\cos\theta_K \, d\cos\theta_l \, d\phi} = \frac{9}{32\pi} \left[ J_{1s} \sin^2 \theta_K + J_{1c} \cos^2 \theta_K + (J_{2s} \sin^2 \theta_K + J_{2c} \cos^2 \theta_K) \cos 2\theta_l \\
+ J_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_K \sin \theta_l \cos \phi \\
+ (J_{6s} \sin^2 \theta_K + J_{6c} \cos^2 \theta_K) \cos \theta_l + J_7 \sin 2\theta_K \sin \theta_l \sin \phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin \phi \\
+ J_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi \right]
$$

Construct th. + exptl. robust observables

$$P_5' = \frac{J_5}{2\sqrt{-J_{2c} J_{2s}}}$$

\[\begin{align*}
&= \sqrt{2} \ \frac{\text{Re}(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*})}{\sqrt{|A_0|^2 \sqrt{|A_\parallel|^2 + |A_\perp|^2}}}
\end{align*}\]

[Descotes-Genon, Matias, Mescia, Ramon, Virto'12]
Exclusive $\bar{B} \to K^* \ell^+ \ell^-$

- Transversity amplitudes ($\lambda = \perp, \parallel, 0$)

\[
A_{L,R}^\lambda = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[ C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}
\]

- Local Form factors:

\[
\mathcal{F}_\lambda^{(T)}(q^2) = \langle K^*_\lambda(k) | \bar{s} \gamma^{(T)} \Gamma b | \bar{B}(k + q) \rangle
\]

- Non-local FFs:

\[
\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x \, e^{iq \cdot x} \langle K^*_\lambda(k) | T \{ J^\mu_{em} (x), C_i Q_i (0) \} | \bar{B}(k + q) \rangle
\]
Exclusive $\bar{B} \rightarrow K^* \ell^+ \ell^-$

- Transversity amplitudes ($\lambda = \perp, ||, 0$)

$$A_{\lambda}^{L,R} = N_{\lambda} \left\{ (C_9 \pm C_{10}) F_{\lambda}(q^2) + \frac{2m_b M_B}{q^2} \left[ C_7 F_{\lambda}^T(q^2) - 16\pi^2 \frac{M_B}{m_b} H_{\lambda}(q^2) \right] \right\}$$

- Local Form factors:

$$F_{\lambda}^{(T)}(q^2) = \langle K^*_\lambda(k) | \bar{s} \Gamma_{\lambda}^{(T)} b | \bar{B}(k + q) \rangle$$

- Non-local FFs:

$$H_{\lambda}(q^2) = i \mathcal{P}_\mu^\lambda \int d^4 x e^{i q \cdot x} \langle K^*_\lambda(k) | T \{ J_{\text{em}}^\mu(x), C_i Q_i(0) \} | \bar{B}(k + q) \rangle$$

- In the heavy-quark limit and at large recoil (large $E_{K^*}$, low $q^2$) the seven $B \rightarrow K^*$ form factors reduce to two universal (soft) form factors.

$$V(q^2) = \frac{m_B + m_{K^*}}{m_B} \xi_\perp(q^2) \quad A_0(q^2) = \frac{E_{K^*}}{m_{K^*}} \xi_{||}(q^2)$$

- Soft form factors cancel in specific angular observables, which then depend on short-distance information only (e.g. $A_T^{(1)}, A_T^{(2)}, P_5', \ldots$)
Local form factors from sum rules (at low $q^2$) and lattice QCD (at high $q^2$)

- For example $V(q^2) \sim F^B_{\perp K^*}(q^2)$

**Sum rules**
- [Bharucha,Straub,Zwicky’15]

**Lattice QCD**
- [Horgan,Liu,Meinel,Wingate’13]
- [Horgan,Liu,Meinel,Wingate’13’15]

**Software**
- EOS v0.2.3
Non-local form factors: OPE + dispersion relation

\[ \mathcal{H}_{\lambda,x}(q^2) = \mathcal{H}_{\lambda,x}^{\text{OPE}}(q_0^2 < 0) + (q^2 - q_0^2) \int_{s_{\text{thr.}}}^{\infty} dt \frac{\rho_{\lambda,x}(t)}{(t - q^2 - i\eta)(t - q_0^2)} \]

\[ \mathcal{H}_{\lambda,x}^{\text{OPE}}(q_0^2 < 0) \] from theory [Khodjamirian, Mannel, Pivovarov, Wang'10'12]

Transition from OPE region to physical region requires data on \( B \rightarrow K^{(*)} X_{1--} \)

- \( \rho_{\lambda,c}(t) : B \rightarrow K^* J/\psi, B \rightarrow K^* \psi(2S), B \rightarrow K^* D\bar{D}, \ldots \)
  - Charm contribution numerically leading

- \( \rho_{\lambda,s}(t) : B \rightarrow K^* \phi, B \rightarrow K^* K\bar{K}, \ldots \)

- \( \rho_{\lambda,ud}(t) : B \rightarrow K^* \rho, B \rightarrow K^* \omega, B \rightarrow K^* \pi\pi, \ldots \)
Non-local form factors

Use conformal variable

\[ z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \]

\[ t_+ = 4M_D^2 \]

\[ t_0 = t_+ - \sqrt{t_+(t_+ - M_{\psi(2S)}^2)} \]

Maps 1st Riemann sheet in \( q^2 \) onto \(|z| < 1\)

Maps \( c\bar{c} \) branch cut in \( q^2 \) onto \(|z| = 1\)

Maps \(-7 \text{ GeV}^2 < q^2 < M_{\psi(2S)}^2\) onto \(|z| < 0.52\)
Global fits to exclusive $b \rightarrow s\ell\ell$

- $C_i^e = C_i^U$
- $C_i^\mu = C_i^U + C_i^V$

$C^V_{9\mu} = -C^V_{10\mu}$
Double differential decay width ($z = \cos \theta_\ell$)

\[
\frac{d^2 \Gamma}{dq^2 \, dz} = \frac{3}{8} \left[ (1 + z^2) H_T(q^2) + 2z H_A(q^2) + 2(1-z^2) H_L(q^2) \right]
\]

Note:

\[
\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2), \quad \frac{dA_{FB}}{dq^2} = \frac{3}{4} H_A(q^2)
\]

- **Low-$q^2$ region**: $1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$
- **High-$q^2$ region**: $q^2 > 14.4 \text{ GeV}^2$
Inclusive $\bar{B} \to X_s \ell^+ \ell^-$

- Dependence of the $H_i$ on Wilson coefficients ($s = q^2/m_b^2$)

\[
H_T(q^2) \propto 2s(1-s)^2 \left| C_9 + \frac{2}{s} C_7 \right|^2 + |C_{10}|^2 \\
H_A(q^2) \propto -4s(1-s)^2 \Re \left[ C_{10} \left( C_9 + \frac{2}{s} C_7 \right) \right] \\
H_L(q^2) \propto (1-s)^2 \left| C_9 + 2 C_7 \right|^2 + |C_{10}|^2
\]

- Normalisation

\[
\frac{d \mathcal{B}(\bar{B} \to X_s ll)}{d \hat{s}} = B_{b \to c e \nu}^{\text{exp.}} \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{1}{C} \frac{d \Gamma(\bar{B} \to X_s ll)/d\hat{s}}{\Gamma(\bar{B} \to X_u e\bar{\nu})}
\]

- LFU ratio

\[
R_{X_s} [q_m^2, q_M^2] \equiv \frac{\int_{q_m^2}^{q_M^2} dq^2 \frac{d \Gamma_{\ell=\mu}}{dq^2}}{\int_{q_m^2}^{q_M^2} dq^2 \frac{d \Gamma_{\ell=e}}{dq^2}}
\]

- High-$q^2$ region, introduce the ratio

\[
\mathcal{R}(s_0) = \frac{\int_{s_0}^{1} d\hat{s} \frac{d \Gamma(\bar{B} \to X_s \ell^+ \ell^-)/d\hat{s}}{\int_{s_0}^{1} d\hat{s} \frac{d \Gamma(\bar{B}^0 \to X_u \ell\nu)/d\hat{s}}{}}}{\int_{s_0}^{1} d\hat{s} \frac{d \Gamma(\bar{B} \to X_s ll)/d\hat{s}}{\Gamma(\bar{B} \to X_u e\bar{\nu})}}
\]

- Normalize to semileptonic $\bar{B}^0 \to X_u \ell\nu$ rate with the same cut

Need differential semi-leptonic $b \to u$ rate
Perturbative and non-perturbative corrections

\[ \Gamma(\bar{B} \rightarrow X_s \ell\ell) = \Gamma(b \rightarrow X_s \ell\ell) + \text{power corrections} \]

- Pert. corrections at quark level are known to NNLO QCD + NLO QED
  
  [Czarnecki, Melnikov, Slusarczyk, Bieri, Ghinculov, Hurth, Isidori, Yao, Greub, Pilipp, Schüpbach, Lunghi, TH]

- Fully differential QCD corrections at NNLO for $Q^9, 10$ also known
  
  [Brucherseifer, Caola, Melnikov’13]

- $1/m_b^2$, $1/m_b^3$ and $1/m_c^2$ non-pert. corrections
  
  [Falk, Luke, Savage’93]
  [Ali, Hiller, Handoko, Morozumi’96]
  [Bauer, Burrell’99; Buchalla, Isidori, Rey’97]

- New in 2020 update
  
  [Hurth, Jenkins, Lunghi, Qin, Vos, TH’20]

- SM prediction of all angular observables + LFU ratio $R_{X_s}$
  
  [Krüger, Sehgal’96]

- More sophisticated implementation of factorizable $c\bar{c}$ contributions via KS approach
  
  [Benzke, Hurth, Fickinger, Turczyk’17-’20]

- Resolved contributions from charm loops

- Monte Carlo study of collinear photon radiation, tailored for Belle II analysis

- Comprehensive model-independent new-physics analysis
SM predictions

- Branching ratio low-$q^2$ region

\[ B[1, 6]_{\mu\mu} = (17.29 \pm 0.76_{\text{scale}} \pm 0.19_{m_t} \pm 0.39_{C,m_c} \pm 0.20_{m_b} \pm 0.09_{\alpha_s} \pm 0.02_{\text{CKM}} \\
\quad \pm 0.26_{\text{BR}_{sl}} \pm 0.12_{\lambda_2} \pm 0.86_{\text{resolved}}) \times 10^{-7} = (17.29 \pm 1.28) \times 10^{-7} \]

- Total error 7.5%, dominated by scale uncertainty and resolved contributions
SM predictions

- Branching ratio low-$q^2$ region

\[ B[1, 6]_{\mu\mu} = (17.29 \pm 0.76_{\text{scale}} \pm 0.19_{m_t} \pm 0.39_{C,m_c} \pm 0.20_{m_b} \pm 0.09_{\alpha_s} \pm 0.02_{\text{CKM}} \]
\[ \pm 0.26_{\text{BR}_l} \pm 0.12_{\lambda_2} \pm 0.86_{\text{resolved}} \times 10^{-7} = (17.29 \pm 1.28) \times 10^{-7} \]

- Total error 7.5%, dominated by scale uncertainty and resolved contributions

- Ratio $R_{X_s}$ has small uncertainty, $R_{X_s}[1, 6] = 0.971 \pm 0.003$
SM predictions

- Branching ratio low-$q^2$ region

\[ B[1, 6]_{\mu\mu} = (17.29 \pm 0.76_{\text{scale}} \pm 0.19_m t \pm 0.39_{C,m c} \pm 0.20_{m b} \pm 0.09_{\alpha_s} \pm 0.02_{\text{CKM}} \\
+ 0.26_{\text{BR}_{sl}} \pm 0.12_{\lambda_2} \pm 0.86_{\text{resolved}}) \times 10^{-7} = (17.29 \pm 1.28) \times 10^{-7} \]

- Total error 7.5%, dominated by scale uncertainty and resolved contributions

- Ratio $R_{\chi s}$ has small uncertainty, \[ R_{\chi s}[1, 6] = 0.971 \pm 0.003 \]

- Branching ratio, high-$q^2$ region

\[ B[> 14.4]_{\mu\mu} = (2.38 \pm 0.27_{\text{scale}} \pm 0.03_m t \pm 0.04_{C,m c} \pm 0.21_{m b} \pm 0.002_{\text{CKM}} \pm 0.04_{\text{BR}_{sl}} \\
+ 0.006_{\alpha_s} \pm 0.12_{\lambda_2} \pm 0.57_{\rho_1} \pm 0.54_{f_{u,s}}) \times 10^{-7} = (2.38 \pm 0.87) \times 10^{-7} \]

- Total error >30%, dominated by HQET annihilation matrix elements
SM predictions

- Branching ratio low-\(q^2\) region

\[
B[1, 6]_{\mu\mu} = (17.29 \pm 0.76_{\text{scale}} \pm 0.19_{m_t} \pm 0.39_{C,m_c} \pm 0.20_{m_b} \pm 0.09_{\alpha_s} \pm 0.02_{\text{CKM}} \\
\quad \pm 0.26_{\text{BR}_{\text{sl}}} \pm 0.12_{\lambda_2} \pm 0.86_{\text{resolved}}) \times 10^{-7} = (17.29 \pm 1.28) \times 10^{-7}
\]

- Total error 7.5%, dominated by scale uncertainty and resolved contributions

- Ratio \(R_{X_s}\) has small uncertainty, \(R_{X_s}[1, 6] = 0.971 \pm 0.003\)

- Branching ratio, high-\(q^2\) region

\[
B[> 14.4]_{\mu\mu} = (2.38 \pm 0.27_{\text{scale}} \pm 0.03_{m_t} \pm 0.04_{C,m_c} \pm 0.21_{m_b} \pm 0.002_{\text{CKM}} \pm 0.04_{\text{BR}_{\text{sl}}} \\
\quad \pm 0.006_{\alpha_s} \pm 0.12_{\lambda_2} \pm 0.57_{\rho_1} \pm 0.54_{f_{u,s}}) \times 10^{-7} = (2.38 \pm 0.87) \times 10^{-7}
\]

- Total error >30%, dominated by HQET annihilation matrix elements

- Different normalisation

\[
R(14.4)_{\mu\mu} = (25.33 \pm 0.27_{\text{scale}} \pm 0.29_{m_t} \pm 0.14_{C,m_c} \pm 0.03_{m_b} \pm 0.07_{\alpha_s} \pm 1.09_{\text{CKM}} \\
\quad \pm 0.04_{\lambda_2} \pm 0.83_{\rho_1} \pm 1.29_{f_{u,s}}) \times 10^{-4} = (25.33 \pm 1.93) \times 10^{-4}
\]

- Total error <10%
Current and projected bounds from inclusive $\bar{B} \to X_s \ell^+ \ell^-$

Current Bounds

Projected Bounds [50 ab$^{-1}$]
Projected bounds using interplay with exclusive $\bar{B} \rightarrow K(\ast)\ell^+\ell^-$ and $\bar{B}_s \rightarrow \mu^+\mu^-$

[Hurth, Jenkins, Lunghi, Qin, Vos, TH'20]

NP constraints
\[ i \frac{d}{dt} \left( \begin{array}{c} |B_q\rangle \\ \bar{B}_q\rangle \end{array} \right) = \left[ \begin{array}{cc} M_{11} & M_{12} \\ M_{12}^* & M_{11} \end{array} \right] - i \frac{1}{2} \left[ \begin{array}{cc} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11} \end{array} \right] \left( \begin{array}{c} |B_q\rangle \\ \bar{B}_q\rangle \end{array} \right) = \left[ M - i \frac{\Gamma}{2} \right] \left( \begin{array}{c} |B_q\rangle \\ \bar{B}_q\rangle \end{array} \right) \]

\begin{itemize}
  \item Diagonalise \( M - i \Gamma/2 \) \( \implies \) eigenvalues \( M_H - i \Gamma_H/2 \) and \( M_L - i \Gamma_L/2 \)
  \item Relate \( |M_{12}^q|, |\Gamma_{12}^q| \) and \( \phi^q = \text{arg}(-M_{12}^q/\Gamma_{12}^q) \) to three observables
    \begin{itemize}
      \item Mass difference: \( \Delta M = M_H - M_L \simeq 2 |M_{12}| \)
      \item Width difference: \( \Delta \Gamma = \Gamma_L - \Gamma_H \simeq 2 |\Gamma_{12}| \cos \phi \)
      \item Flavour-specific CP asymmetry
    \end{itemize}
  \item \( a_{fs}^q \) measured from semileptonic decays, small in SM
\end{itemize}
Mass difference

- Many applications
  - CKM elements, $|V_{td}/V_{ts}|$, $|V_{ts}V_{tb}|$, ...
  - UT angles, BSM constraints

- In the SM
  \[ M_{12}^s = \frac{G_F^2}{12\pi^2} \chi_t^2 \, M_W^2 \, S_0(x_t) \, B(\mu) \, f_{B_s}^2 \, M_{Bs} \, \hat{\eta}_B \]

- Extraction of decay constant $f_{B_s}$ and bag parameter $B(\mu)$ from
  - HQET Sum rules: Mannel et al. (2016-18), Lenz et al. (2017)
- Same analysis for $B_s$ system

This work: $\xi = f_{B_s}^2 f_{B_d}^2 \frac{1}{\sqrt{B_{B_s}^1}} \frac{1}{\sqrt{B_{B_d}^1}} = 1.200 \pm 0.0054 \pm 0.0060$
Mass difference

- Same analysis for $B_s$ system

- Very precise prediction of ratio
  \[
  \xi = \frac{f_{B_s}}{f_{B_d}} \frac{\sqrt{B_{1s}}}{\sqrt{B_{1d}}}
  = 1.200^{+0.0054}_{-0.0060}
  \]

\[\text{Average:}\]
- HPQCD'19
- KLR'19
- RBC/UKQCD'18
- FLAG'19 (2+1)
- GMP'17
- FNAL/MILC'16
- ETMC'13

[Di Luzio,Kirk,Lenz,Rauh'19]
Mass difference

- Average of lattice + sum rules

\[
\Delta M_d \text{Average 2019} = \left( 0.533^{+0.022}_{-0.036} \right) \text{ps}^{-1} = \left( 1.05^{+0.04}_{-0.07} \right) \Delta M_d^{\text{exp}}
\]

\[
\Delta M_s \text{Average 2019} = \left( 18.4^{+0.7}_{-1.2} \right) \text{ps}^{-1} = \left( 1.04^{+0.04}_{-0.07} \right) \Delta M_s^{\text{exp}}.
\]

- Good agreement with expt.

\[
\left| \frac{V_{td}}{V_{ts}} \right| = 0.2043^{+0.0010}_{-0.0011}
\]

- Slightly below CKMfits

- Prediction for ratio of mass differences

\[
\frac{\Delta M_d}{\Delta M_s} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{1}{\xi^2} \frac{M_{B_d}}{M_{B_s}} = 0.0298^{+0.0005}_{-0.0009}
\]

- Agreement at 1.4\(\sigma\) with experiment: \((\Delta M_d/\Delta M_s)\text{exp.} = 0.0285 \pm 0.0001\)
Assume $\delta C_9^\mu = -\delta C_{10}^\mu$. Simplified $Z'$ and scalar leptoquark model.

Avg. '19 (2\sigma excl.)  FLAG '19 (2\sigma excl.)  Future '25 (2\sigma excl.)  2% non-pert./1% $V_{cb}$
Lifetimes, recent progresses

- Dimension-six matrix elements from sum rules
  - Calculation proceeds along the lines of mass-difference calculation

![Graph showing lifetime ratios](image-url)

[Kirk, Lenz, Rauh'17]

[Graph showing lifetime ratios](image-url)

[Nierste et al.'17'20]

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Precision flavour physics
Lifetimes, recent progresses

- Dimension-six matrix elements from sum rules
  - Calculation proceeds along the lines of mass-difference calculation

- Calculation of $\Gamma_{12}^q$ and $a_{fs}^q$ at order $O(\alpha_s^2 N_f)$
  - Scale dependence still rather large
  - Significantly reduced scheme dependence
Dimension-six matrix elements to Darwin $\rho_D$ and Spin-orbit $\rho_{LS}$ term in $\tilde{\Gamma}_3$

[Mannel, Pivovarov, Moreno’20; Lenz, Piscopo, Rusov’20]

- Compute matching coefficients $C_{\rho_D}$ and $C_{\rho_{LS}}$

$$\Gamma(b \to c\bar{q}_1q_2) = \Gamma^0 \left[ C_0 - C_{\mu\pi} \frac{\mu_\pi^2}{2m_b^2} + C_{\mu_G} \frac{\mu_G^2}{2m_b^2} - C_{\rho_D} \frac{\rho_D^3}{2m_b^3} - C_{\rho_{LS}} \frac{\rho_{LS}^3}{2m_b^3} \right]$$
Lifetimes

- Dimension-six matrix elements to Darwin $\rho_D$ and Spin-orbit $\rho_{LS}$ term in $\tilde{\Gamma}_3$
  
  \[ [\text{Mannel, Pivovarov, Moreno'20; Lenz, Piscopo, Rusov'20}] \]

- Compute matching coefficients $C_{\rho_D}$ and $C_{\rho_{LS}}$

\[
\Gamma(b \rightarrow c\bar{q}_1q_2) = \Gamma^0 \left[ C_0 - C_{\mu\pi} \frac{\mu^2}{2m_b^2} + C_{\mu G} \frac{\mu^2}{2m_b^2} - C_{\rho_D} \frac{\rho_D^3}{2m_b^3} - C_{\rho_{LS}} \frac{\rho_{LS}^3}{2m_b^3} \right]
\]

- Combine with non-leptonic tree-level decays
  
  \[ [\text{Tetlalmatzi, Lenz'19}] \]
Amplitude in the limit $m_b \gg \Lambda_{\text{QCD}}$

\[
\langle M_1 M_2 | Q_i | B \rangle \simeq m_B^2 F_{+}^{B\to M_1} (0) f_{M_2} \int_0^1 du \ T_I^I (u) \ \phi_{M_2} (u) + (M_1 \leftrightarrow M_2)
+ f_B f_{M_1} f_{M_2} \int_0^\infty d\omega \int_0^1 dv du \ T_{II}^{II} (\omega, v, u) \ \phi_B (\omega) \ \phi_{M_1} (v) \ \phi_{M_2} (u) + \mathcal{O} \left( \frac{\Lambda_{\text{QCD}}}{m_b} \right)
\]

- $T_{I,II}$: Hard scattering kernels, perturbatively calculable

- $F_+ : B \to M$ form factor

- $f_i : \text{decay constants}$

- $\phi_i : \text{light-cone distribution amplitudes}$

Strong phases are $\mathcal{O}(\alpha_s)$ and/or $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$
Classification of amplitudes

- $\alpha_1$: colour-allowed tree amplitude
  
- $\alpha_2$: colour-suppressed tree amplitude
  
- $\alpha_{u,c}^4$: QCD penguin amplitudes

$$
\sqrt{2} \langle \pi^- \pi^0 | \mathcal{H}_{\text{eff}} | B^- \rangle = A_{\pi\pi} \lambda_u \left[ \alpha_1(\pi\pi) + \alpha_2(\pi\pi) \right]
$$

$$
\langle \pi^+ \pi^- | \mathcal{H}_{\text{eff}} | \bar{B}^0 \rangle = A_{\pi\pi} \left\{ \lambda_u \left[ \alpha_1(\pi\pi) + \alpha_{u}^4(\pi\pi) \right] + \lambda_c \alpha_{c}^4(\pi\pi) \right\}
$$

$$
- \langle \pi^0 \pi^0 | \mathcal{H}_{\text{eff}} | \bar{B}^0 \rangle = A_{\pi\pi} \left\{ \lambda_u \left[ \alpha_2(\pi\pi) - \alpha_{u}^4(\pi\pi) \right] - \lambda_c \alpha_{c}^4(\pi\pi) \right\}
$$

$$
\langle \pi^- K^0 | \mathcal{H}_{\text{eff}} | B^- \rangle = A_{\pi K} \left[ \lambda^{(s)}_u \alpha_{u}^4 + \lambda^{(s)}_c \alpha_{c}^4 \right]
$$

$$
\langle \pi^+ K^- | \mathcal{H}_{\text{eff}} | \bar{B}^0 \rangle = A_{\pi K} \left[ \lambda^{(s)}_u (\alpha_1 + \alpha_{u}^4) + \lambda^{(s)}_c \alpha_{c}^4 \right]
$$

- Tree amplitudes $\alpha_1$ and $\alpha_2$ known analytically to NNLO [Bell'07'09; Beneke,Li,TH'09]
Penguin amplitudes at two loops

- Contributing two-loop penguin diagrams

- Plus: $Q_{3-6}$ insertion into tree topology (66 diagrams, known)
  and one-loop diagrams involving $Q_8$ (18 diagrams, simple)
Kinematics:

\[ p^2 = q^2 = 0 \]
\[ p_b^2 = m_b^2 \]

- Fermion loop with either \( m = 0 \) or \( m = m_c \).
- Genuine two-scale problem: \( \bar{u}, \frac{m_c^2}{m_b^2} \)
- Threshold at \( \bar{u} = 4 \frac{m_c^2}{m_b^2} \)
- Choice of suitable kinematic variables crucial

\[
\begin{align*}
    s &= \sqrt{1 - 4 z_c / \bar{u}} \quad r = \sqrt{1 - 4 z_c} \\
    \bar{u}, z_c &= \frac{m_c^2}{m_b^2} \quad s_1 = \sqrt{1 - 4 / \bar{u}} \quad r \\
    p &= \frac{1 - \sqrt{u^2 + 4 u z_c}}{u} \quad r
\end{align*}
\]
Two-loop calculation

- Regularize UV and IR divergences dimensionally. Poles up to $1/\epsilon^3$
- Reduction: IBP relations, Laporta algorithm in FIRE, get 36 master integrals
  [Tkachov’81; Chetyrkin,Tkachov’81] [Laporta’01; Smirnov’08]
- Use differential equations in canonical form
  \[ d \tilde{M}(\epsilon, x_n) = \epsilon d\tilde{A}(x_n) \tilde{M}(\epsilon, x_n) \]
  [Henn’13]
- Analytic solution in terms of iterated integrals (GPLs) over alphabet
  \[ \{ 0, \pm 1, \pm 3, \pm i\sqrt{3}, \pm r, \pm \frac{r^2 + 1}{2}, \pm (1 + 2\sqrt{z_c}), \pm (1 - 2\sqrt{z_c}) \} \]
  [Bell,TH’14]
- Catalyses analytic convolution with LCDA
- UV renormalisation, IR subtraction, matching onto SCET, pole cancellation
- Convolution with LCDA
  \[ \int_0^1 du \ T_i(u) \ \phi_M(u) \]
  - Expand LCDA of light meson in Gegenbauer polynomials
    \[ \phi_M(u) = 6u\bar{u} \left[ 1 + \sum_{n=1}^{\infty} a_n^M C_n^{(3/2)} (2u - 1) \right] \]
Canonical basis for master integrals

\[
\frac{M_{18}}{u\epsilon^3} = \begin{array}{c}
\end{array} + \begin{array}{c}
\end{array}
\]

\[
\frac{M_{19}}{u\epsilon^3} = \begin{array}{c}
\end{array} + \begin{array}{c}
\end{array}
\]

\[
\frac{2M_{20}}{u\epsilon^3} = \begin{array}{c}
\end{array} + \begin{array}{c}
\end{array}
\]

\[
\frac{M_{21}}{\epsilon^2} = 2\frac{[(1 + \bar{u})^2 z_c - \bar{u}^2]}{\bar{u}} + \bar{u}s^2(1 + \bar{u}) + \frac{2\epsilon u}{m_b^2} + \begin{array}{c}
\end{array} + \begin{array}{c}
\end{array}
\]

\[
\frac{dM_{19}}{ds} = \frac{4\epsilon M_{18} r (r^2 + 1)}{((r^2 + 1)^2 - 4s^2)} - \frac{2\epsilon M_{19} r (r^2 + s^2 - 2)}{(1 - r^2)(r^2 - s^2)} + \frac{4\epsilon M_{20} r s}{((r^2 + 1)^2 - 4s^2)} - \frac{\epsilon M_{21} r (r^2 + 1)}{((r^2 + 1)^2 - 4s^2)}
\]

\[
\text{Boundary conditions}
\]

- \(M_{18}\) and \(M_{19}\) vanish in \(s = r\) (i.e. in \(u = 0\))
- \(M_{20}\) and \(M_{21}\) vanish in \(s = +i\infty\) (i.e. in \(u = 1\))
Penguin amplitudes: Anatomy of corrections

\[ a_4^P[\pi K] \]

\[ a_4^C[\pi K] \]

LO

NNLO\_Q_{3-6,8g}

NNLO\_Q_{1,2}

NLO

NNLO\_\text{full}

Im[\alpha_4^P]

Re[\alpha_4^P]

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Precision flavour physics
Results

- Penguin amplitudes: Scale dependence.
  - Only form factor term, no spectator scattering.

\[ a_4^u \]

\[ a_4^c \]

Orange: LO (dashed) and NLO (solid)  
Blue: NNLO \( Q_{1,2} (d), Q_{3-6,8g} (dd), \) all (s)

[Bell,Beneke,Li,TH’20]
Direct CP asymmetries in percent.
Errors are CKM and hadronic, respectively.

<table>
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<th>$f$</th>
<th>NLO</th>
<th>NNLO</th>
<th>NNLO + LD</th>
<th>Exp</th>
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LCDA of the $B$-meson

Parton picture: 2-particle Fock state

- Some external light-like momentum, e.g. $p_{\gamma}^\mu = E_\gamma n^\mu$, $n^2 = 0$
- with $\omega \equiv n \cdot k$ light-cone projection of light antiquark momentum
- $\phi_B(\omega)$ as probability amplitude

Field theoretical definition of $\phi_B^+(\omega)$ from light-cone operators in HQET:

$$m_B f_B^{(HQET)} \phi_B^+(\omega) = \int \frac{d\tau}{2\pi} e^{i\omega \tau} \langle 0 | \bar{q}(\tau n) [\tau n, 0] \gamma_5 h_{v}^{(b)}(0) | \bar{B}(m_B v) \rangle$$

[Grozin,Neubert'96]
LCDA of the $B$-meson

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- $\phi_B(\omega)$ as probability amplitude

Field theoretical definition of $\phi^+_B(\omega)$ from light-cone operators in HQET:

$$m_B f_B^{(HQET)} \phi^+_B(\omega) = \int \frac{d\tau}{2\pi} e^{i\omega \tau} \langle 0| \bar{q}(\tau n) [\tau n, 0] \gamma_5 h_v^{(b)}(0) | \bar{B}(m_B v) \rangle$$

[Grozin,Neubert'96]

- in QCD factorization theorems one encounters logarithmic moments

$$L_n(\mu) = \int_0^\infty \frac{d\omega}{\omega} \ln^n \left( \frac{\omega}{\mu} \right) \phi^+_B(\omega) \quad (n=0,1,2, \ldots)$$

- in QCD light-cone sum rules, one is sensitive to low light-cone momenta

$$\phi^+_{B'}(0), \quad \phi^+_{B''}(0), \quad \text{etc.}$$
Theoretical developments

- 1-loop RGE
  - [Lange, Neubert'03]

- Radiative tail including dim-5 HQET operators
  - [Kawamura, Tanaka'08]

\[ \tilde{\phi}^+ (\tau) \xrightarrow{\tau \to 0} 1 - \frac{\alpha_s C_F}{4\pi} \left( 2 \ln^2 (i\tau \mu) + 2 \ln (i\tau \mu) + \frac{5\pi^2}{12} \right) + \mathcal{O}(\tau) + \mathcal{O}(\tau^2) \]

- Eigenfunctions of 1-loop RGE
  - [Bell, Feldmann, Wang'13; Braun, Manashov'14]

\[ \phi^+_B (\omega; \mu) = \int_0^\infty ds \sqrt{\omega s} J_1(2\sqrt{\omega s}) U(s; \mu, \mu_0) \eta^+_B(s; \mu_0) \]

- 2-loop RGE from conformal symmetry
  - [Braun, Ji, Manashov'19]

\[ \left( \frac{d}{d \ln \mu} + \Gamma_c \ln (\mu \, s \, e^{2\gamma_E}) + \gamma_+ \right) \eta^+_B(s, \mu) = 4 C_F \left( \frac{\alpha_s}{4\pi} \right)^2 \int_0^1 du \frac{\bar{u}}{u} h(u) \eta^+_B(\bar{u}s, \mu) \]

- 2-loop evolution for generating function of log. moments
  - [Galda, Neubert'20]
Simplest application: \( B \rightarrow \gamma \) form factors

Consider \( \bar{B} \rightarrow \gamma \ell \nu \)

For large photon energy, \( E_\gamma \sim m_b/2 \):

\[
(p_\gamma - p_\bar{u})^2 \simeq -2 \, p_\gamma \cdot p_\bar{u} \equiv -2 \, E_\gamma \omega
\]

Sensitive to light-cone projection \( \omega \) of light antiquark momentum in \( B \)-meson

\[
F^{B\rightarrow\gamma}(E_\gamma) \simeq [\text{kinematic factor}] \times \int_0^\infty \frac{d\omega}{\omega} \phi_B(\omega)
\]

Experimental bound on BR \( \rightarrow \) bound on \( \frac{1}{\lambda_B} \equiv \int_0^\infty \frac{d\omega}{\omega} \phi_B(\omega) \)

[Beneke,Rohrwild’11; Braun,Khodjamirian’12; Beneke,Braun,Wei’18]
Standard Model effective field theory (SMEFT)

Basis of dim.-6 operators made of SM fields and with SM gauge symmetry

Approx. 60 operators, proliferates to $\sim 2500$ with flavour indices

Look at various subsets of Wilson coefficients, find constraints with global fits

Scenario: only four non-zero SMEFT Wilson coefficients at 2 TeV

Solid (dashed) contours include (exclude) Moriond-2019 results for $R_K$, $R_{K^*}$, $R_D$, and $R_{D^*}$

[Aebischer, Altmannshofer, Guadagnoli, Reboud, Stangl, Straub’19]
Many of the yet unanswered questions of particle physics are related to the Yukawa sector of the SM.

Flavour sector of the SM is currently being investigated to unprecedented precision.

Description of quark flavour sector benefits from interplay of many different aspects, many of which are also important in other branches of particle physics.
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Exciting times in particle physics are ahead of us

Flavour physics will help to (hopefully) reveal and quantify the remaining mysteries in particle physics