Top quark mass at the LHC

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RWTH Aachen University
This talk is about issues surrounding the top-quark mass definition

The particular emphasis on some of the issues is due to performing the measurement at the LHC

This talk is for non-experts

The topic is controversial
Status and prospects
Why am I talking about this?
(almost)
every statement in physics
is only approximate
• The Operator Product Expansion (used to decompose observables into non-perturbative and perturbative factors) does not converge

• The perturbative expansion of a partonic cross section does not converge

• The relation between the on-shell and short-distance masses of a quark does not converge

We are only optimizing asymptotic series
Renormalons in the relation between on-shell and $\overline{\text{MS}}$ mass

$$m_{\text{pole}} = m_{\overline{\text{MS}}} + \mu C_F \frac{\alpha_S}{\pi} e^{-C/2} \sum_{n=0}^{\infty} 2^n n! (a_S b_0)^n$$

Bigi, Shifman, Uraltsev, Vainstein '94
Beneke, Braun '94

Ambiguity estimate: $1/\pi$ of imaginary part of Borel transform

$$\delta m = \mu 2 C_F \frac{e^{-C/2}}{b_0} e^{-1/2 a_S b_0}$$

$$a_S(\mu^2) = \frac{1}{b_0 \ln \mu^2 / \Lambda_{\text{QCD}}^2}$$

$$\delta m = 2 C_F \frac{e^{-C/2}}{b_0} \Lambda_{\text{QCD}}$$

Deeper understanding using renormalization group methods

Beneke '94
Renormalons in the relation between on-shell and $\overline{\text{MS}}$ mass

Relation to $\overline{\text{MS}}$ mass up to 4-loops

$$m_p = 163.643 + 7.557 + 1.617 + 0.501 + (0.195 \pm 0.005) \text{ GeV}$$

Beneke, Marquard, Nason, Steinhauser ’16

Recent estimate of the ambiguity

$$\delta^{(5+)} m_p = 0.304^{+0.012}_{-0.063} (N) \pm 0.030 (m_{b,c}) \pm 0.009 (\alpha_s) \pm 0.108 \text{ (ambiguity)} \text{ GeV}$$

…Countered with an estimate of ambiguity at 250 MeV
Hoang, Lepenik, Preisser ’17

This ambiguity is not the crux of the problem
A finite top-quark width does not solve the renormalon problem

\[ \sum^{(1)} = \frac{t}{(a)} + \frac{b}{(b)} \]

\[ \sum_{n=0}^{\infty} \frac{t}{(a')} \]

Figure 3: Diagrams contributing to the top-quark self-energy at leading order in \( \alpha_s \) and \( \alpha_W \). Fig. (a') replaces Fig. (a) when summing to all orders in \( \beta_0 \alpha_s \).

Smith, Willenbrock '96
A finite top-quark width does not solve the renormalon problem.

\[ \Sigma = \]

\[ \sum_{n=0}^{\infty} \]

Figure 4: Diagrams contributing to the top-quark self energy at leading order in \( \alpha_s \), but to all orders in \( \alpha_W \). Fig. (a') replaces Fig. (a) when summing to all orders in \( \beta_0 \alpha_s \).
A quark does not have a pole mass

Figure 1: A scattering amplitude factorizes when an internal propagator is near its pole. The external lines represent color-singlet asymptotic states.

Figure 2: The production and decay of a top quark in (a) perturbation theory, and (b) nonperturbatively.

Smith, Willenbrock '96
No infrared region in definition of quark mass improves convergence

Figure 1: The total normalised photon-induced $t\bar{t}$ cross section $R$ at the LC versus the c.m. energy in the threshold regime at LO (dotted curves), NLO (dashed) and NNLO (solid) in the pole mass scheme for $M_{\text{pole}}^T = 175.05$ GeV, $\alpha_s(M_{Z}) = 0.119$, $\Gamma_t = 1.43$ GeV and $\mu_{\text{soft}} = 15, 30, 60$ GeV. The plots have been generated from results provided by the groups Hoang-Teubner (HT), Melnikov-Yelkhovsky-Yakovlev-Nagano-Ota-Sumino (MYNOS), Penin-Pivovarov (PP) and Beneke-Signer-Smirnov (BSS).

Figure 2: The total normalised photon-induced $t\bar{t}$ cross section $R$ at the LC versus the c.m. energy in the threshold regime at LO (dotted curves), NLO (dashed) and NNLO (solid). Hoang-Teubner used the IS mass scheme with $m_t^{IS} = 173.68$ GeV, Melnikov-Yelkhovsky the kinetic mass at 15 GeV with $m_t^{IS,15\text{GeV}} = 173.10$ GeV, and Beneke-Signer-Smirnov and Yakovlev the PS mass at 20 GeV with $m_t^{IS,20\text{GeV}} = 173.30$ GeV. The plots have been generated from results provided by the groups Hoang-Teubner (HT), Melnikov-Yelkhovsky (MY) and Beneke-Signer-Smirnov (BSS) and Yakovlev.

Hoang, Beneke, Melnikov, Nagano, Ota, Penin, Pivovarov, Signer, Smirnov, Sumino, Teubner, Yakovlev, Yelkhovsky `00
The $\overline{\text{MS}}$ mass is not as universal as the $\overline{\text{MS}}$ strong coupling.

Table 1: The various contributions to $m_t(M_t^2) - M_t$ in GeV.

<table>
<thead>
<tr>
<th>$M_H$ [GeV]</th>
<th>$O(\alpha)$</th>
<th>$O(\alpha \alpha_s)$</th>
<th>$O(\alpha) + O(\alpha \alpha_s)$</th>
<th>$O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3)$</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>124</td>
<td>12.11</td>
<td>-0.39</td>
<td>11.72</td>
<td>-10.38</td>
<td>1.34</td>
</tr>
<tr>
<td>125</td>
<td>11.91</td>
<td>-0.39</td>
<td>11.52</td>
<td>-10.38</td>
<td>1.14</td>
</tr>
<tr>
<td>126</td>
<td>11.71</td>
<td>-0.38</td>
<td>11.32</td>
<td>-10.38</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Jegerlehner, Kalmykov, Kniehl `12
Renormalons are a proxy for non-perturbative effects

First discussed in the context of soft and collinear radiation for the thrust distribution in electron—positron collisions

Gardi, Rathsman `01

and higher-twist contributions to deep inelastic scattering

Gardi, Roberts `02
Renormalons are a proxy for non-perturbative effects

Recently revisited in Ravasio, Nason, Oleari `18

(a) \( \bar{b} \quad t \quad b \quad W^* \)

(b) \( \bar{b} \quad t \quad b \quad W^* \)

(c) \( \bar{b} \quad q \quad \bar{q} \quad W^* \)

(d) \( \bar{b} \quad t \quad b \quad W^* \)
Renormalons are a proxy for non-perturbative effects

Recently revisited in Ravasio, Nason, Oleari ‘18

1. total cross section: no infra-red sensitivity if the result is written in terms of short-distance masses; gets reflected in the convergence of perturbative series;

2. jet selection introduces linear infra-red sensitivity (1/R); independent of the mass definition used; this is known and anticipated effect; the jet energy/momentum is affected by the hadronization in that way;

3. W-b invariant mass has linear infra-red sensitivity due to a final state jet, irrespective of the mass definition used;

4. Average energy of the W-boson: exhibits linear infra-red sensitivity in the narrow width limit, regardless of the mass used (boost to top rest frame); however, if the calculation is done keeping the width of the top quark, there is no linear infra-red sensitivity in case the short-distance mass is used.
Modelling at hadron colliders

Parton showers matched after decay with NLO precision
Ravasio, Jezo, Nason, Oleari `18

Table 1: Differences in the $m_{W_{b_j}}^{\text{max}}$ for $m_t=172.5$ GeV for $t\bar{t}dec$ and $hvq$ with respect to $b\bar{b}4\ell$, showered with Pythia8.2, at the NLO+PS level and at the full hadron level. Results obtained after smearing the $m_{W_{b_j}}$ distribution with a Gaussian function with a 15 GeV width are also shown in order to mimic effects due to experimental uncertainties.

<table>
<thead>
<tr>
<th></th>
<th>PS only</th>
<th>full</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No smearing</td>
<td>smearing</td>
</tr>
<tr>
<td>$b\bar{b}4\ell$</td>
<td>172.522 GeV</td>
<td>171.403 GeV</td>
</tr>
<tr>
<td>$t\bar{t}dec - b\bar{b}4\ell$</td>
<td>$-18 \pm 2$ MeV</td>
<td>$+191 \pm 2$ MeV</td>
</tr>
<tr>
<td>$hvq - b\bar{b}4\ell$</td>
<td>$-24 \pm 2$ MeV</td>
<td>$-89 \pm 2$ MeV</td>
</tr>
</tbody>
</table>

hvq:
NLO production

tTdec:
NLO production and decay, narrow width

bB4l:
full NLO, off-shell
Modelling at hadron colliders

Parton showers matched after decay with NLO precision
Ravasio, Jezo, Nason, Oleari `18

$\mu^e_q$:
NLO production

tTdec:
NLO production and decay, narrow width

bB4l:
full NLO, off-shell

<table>
<thead>
<tr>
<th></th>
<th>No smearing</th>
<th>15 GeV smearing</th>
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<tbody>
<tr>
<td></td>
<td>He7.1</td>
<td>He7.1</td>
</tr>
<tr>
<td>$\bar{b}b\mu\ell$</td>
<td>172.727 GeV</td>
<td>171.626 GeV</td>
</tr>
<tr>
<td></td>
<td>+66 ± 7 MeV</td>
<td>+1091 ± 2 MeV</td>
</tr>
<tr>
<td>$t\bar{t}dec$</td>
<td>172.775 GeV</td>
<td>171.678 GeV</td>
</tr>
<tr>
<td></td>
<td>+39 ± 5 MeV</td>
<td>+1179 ± 2 MeV</td>
</tr>
<tr>
<td>$hvq$</td>
<td>173.038 GeV</td>
<td>172.319 GeV</td>
</tr>
<tr>
<td></td>
<td>-235 ± 5 MeV</td>
<td>+251 ± 2 MeV</td>
</tr>
</tbody>
</table>

Table 2: $m_{Wb\ell}$ peak position for $m_t=172.5$ GeV obtained with the three different generators, showered with Herwig7.1 (He7.1). The differences with Pythia8.2 (Py8.2) are also shown.
Modelling at hadron colliders

Observable used for a recent top quark mass determination

Off-shell, NWA

\[ pp \rightarrow t\bar{t}j \rightarrow e^+\nu_e \mu^-\bar{\nu}_\mu b\bar{b} j @ 13 \text{ TeV LHC} \]

\[ \mathcal{R}(m_t^{\text{pole}}, \rho_s) = \frac{1}{\sigma_{t\bar{t}j}} \left( \frac{d\sigma_{t\bar{t}j}}{d\rho_s} \right)(m_t^{\text{pole}}, \rho_s) \]

\[ \rho_s = \frac{2m_0}{M_{t\bar{t}j}} \]

Alioli, Fernandez, Fuster, Irles, Moch, Uwer, Vos ‘13

Bevilacqua, Hartanto, Kraus, Schulze, Worek ‘18
Modelling at hadron colliders

**pp → t¯ tj → e⁺νe μ⁻νμ b¯ bj @ 13 TeV LHC**

<table>
<thead>
<tr>
<th>Theory, NLO QCD</th>
<th>$m_t^{\text{out}} \pm \delta m_t^{\text{out}}$ [GeV]</th>
<th>Averaged $\chi^2$/d.o.f.</th>
<th>Probability $p$-value</th>
<th>$m_t^{\text{in}} - m_t^{\text{out}}$ [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CT14 PDF</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full, $\mu_0 = H_T/2$</td>
<td>173.38 ± 1.34</td>
<td>1.04</td>
<td>0.40 (0.8σ)</td>
<td>−0.18</td>
</tr>
<tr>
<td>Full, $\mu_0 = E_T/2$</td>
<td>172.84 ± 1.33</td>
<td>1.05</td>
<td>0.39 (0.9σ)</td>
<td>+0.36</td>
</tr>
<tr>
<td><strong>Full, $\mu_0 = m_t$</strong></td>
<td>174.11 ± 1.39</td>
<td>1.07</td>
<td>0.37 (0.9σ)</td>
<td>−0.91</td>
</tr>
<tr>
<td>NWA, $\mu_0 = m_t$</td>
<td>175.70 ± 0.96</td>
<td>1.17</td>
<td>0.24 (1.2σ)</td>
<td>−2.50</td>
</tr>
<tr>
<td>NWA$_{\text{Prod.}}, \mu_0 = m_t$</td>
<td>169.93 ± 0.98</td>
<td>1.20</td>
<td>0.20 (1.3σ)</td>
<td>+3.27</td>
</tr>
<tr>
<td><strong>31 bins</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>5 bins</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full, $\mu_0 = H_T/2$</td>
<td>173.15 ± 1.32</td>
<td>0.93</td>
<td>0.44 (0.8σ)</td>
<td>+0.05</td>
</tr>
<tr>
<td>Full, $\mu_0 = E_T/2$</td>
<td>172.55 ± 1.18</td>
<td>1.07</td>
<td>0.37 (0.9σ)</td>
<td>+0.65</td>
</tr>
<tr>
<td><strong>Full, $\mu_0 = m_t$</strong></td>
<td>173.92 ± 1.38</td>
<td>1.48</td>
<td>0.20 (1.3σ)</td>
<td>−0.72</td>
</tr>
<tr>
<td>NWA, $\mu_0 = m_t$</td>
<td>175.54 ± 0.97</td>
<td>1.38</td>
<td>0.24 (1.2σ)</td>
<td>−2.34</td>
</tr>
<tr>
<td>NWA$_{\text{Prod.}}, \mu_0 = m_t$</td>
<td>169.37 ± 1.43</td>
<td>1.16</td>
<td>0.33 (1.0σ)</td>
<td>+3.83</td>
</tr>
</tbody>
</table>

Bevilacqua, Hartanto, Kraus, Schulze, Worek ‘18
Modelling at hadron colliders

- Output of a measurement using Monte Carlo event generators is function of the pole mass

\[ m_{t}^{MC} = m_{t}^{pole} + \Delta_{pert}^{m} + \Delta_{non-pert}^{m} + \Delta_{MC}^{m} \]

- Study the effects by comparison of an event generator and analytic QCD predictions in a controlled environment

- For example lepton collider in boosted topologies

Hoang et al. (ongoing)
Modelling at hadron colliders

- One source of shifts may be due to radiation cutoffs

\[ \rightarrow \text{Coherent branching: (basis of the Herwig parton shower)} \]

\[ \mu^2 = p_{\perp}^2 + (1 - z)^2 m^2 \text{ \quad cutoff: } p_{\perp}^2 > Q_0^2 \]

\[ m_{t}^{\text{CB}}(Q_0) = m_{t}^{\text{pole}} - \frac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s(Q_0)^2) \]

"Coherent-branching mass" is here the "Monte Carlo mass"

Herwig 7: \( Q_0 = 1.25 \text{ GeV} \quad \rightarrow \quad m_{t}^{\text{Herwig}} = m_{t}^{\text{CB}}(1.25 \text{ GeV}) \)

Hoang, Plätzter, Samitz `18

Catani, Marchesini, Webber 1991
Gieseke, Stephens, Webber, 2003

Usually not present in analytic QCD!
Modelling at hadron colliders

- Unanswered question: does this effect persist if matching after decay?

- Further restrictions of the analysis
  - Boosted top quarks
  - Narrow width approximation
  - Top production (2-jettiness)

- Needed to remove the restrictions
  - Parton shower algorithm for slow tops
  - Parton shower for unstable top quark
  - Factorized predictions including top decay
Conclusions

- Direct measurements of the top-quark mass can be interpreted in terms of the pole mass

- While not necessary at present, it is possible to directly determine a short distance mass
  
  - This is easy for specific observables by re-expanding theory predictions in the on-shell scheme in terms of a different mass parameter
  
  - This is difficult using parton showers, since the underlying differential fixed-order prediction is given in function of the on-shell mass

- Ambiguities of parton shower modelling may and should be studied further