Talk at LoopFest XX

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Slide one

Hadronic production of top quarks at LHC provides an opportunity to study the heaviest particle of the Standard Model in great details. It leads to a better understanding of electroweak symmetry breaking as the mass of the top comes exclusively through interactions with the Higgs background field. The top quark is primarily produced in pairs, but single-top production also occurs frequently. The total cross-section for single-top production is approximately one quarter of the one for pair production. As single-top production involves electroweak interactions, it allows a direct probation of the Cabbibo-Komayachi-Maskawa matrix element Vbt. Consequently, it allows an indirect determination of the decay rate of the top-quark and its mass. Finally, it can constain the bottom-quark parton distribution function.

Slide two

The single-top production can be split into three modes: the s channel, presented to the right of the slide, is the least frequent production mode. The associated production where a W boson is present in the final state and the t channel which is the privileged way to produce single top-quark at LHC. The t-channel will be the subject of this talk.

Slide three

A precise theoretical description of the t-channel single-top production can be obtain in the context of perturbative QCD. Both QCD and electroweak NLO corrections are know since more than ten years now. The NNLO corrections are known with top-quark decay and QCD corrections to it in the narrow-width approximation. However, all existing calculations exclude the non-factorisable contribution. These are made of corrections connecting the two quark lines whereas factorisable corrections consists on corrections on the same quark line.

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If we neglect the non-factorisable correction, we work in the factorisable approximation. The non-factorisable corrections are colour suppressed and therefore and expected to be negligible compared to the factorisable corrections. We indeed see on these two graphs that the suppression amounts to a factor $N_c^2 - 1 = 8$. In addition, we cannot rely on the NLO to have any indication of the contribution of these peculiar corrections since the corrections vanish due to colour conservation.

Slide five

However, it is not clear that the non-factorisable contributions are indeed negligible. First of all, it appears that the factorisable NNLO corrections to the NLO cross-section are actually small; less than 1%. In addition, non-factorsiable corrections could be enhanced by a factor π^2 due to the Glauber phase. This is a virtual effect that, in principle, does not require a scattering to occur. It happens therefore as the transverse momentum of the top quark goes to zero. I despict the effect on these equation which is an expansion of the total cross-section in $p_\perp^t$ over the partonic energy. This is a valid expansion as, in this process, the tranverse momentum of the top amounts to 40 GeV and the partonic energy to 300 GeV. If it exists in this process, the π^2 enhancement affects the term σ_0. From this equation, we can also forsee that the virtual cross section is expected to be dominant in this process. Indeed, it can be argue that, in this limit where $p_\perp^t$ goes to zero, real emission amplitude do not contribute to σ_0 and therefore, are suppressed by a factor $p_\perp^t/\sqrt{s}$. We note that this enhancement from the Glauber phase has explcitely been proved for the non-factorisable corrections to the Higgs production in weak boson fusion in the eikonal approximation. In our case, this factor, close to 10, could clearly compensate the colour suppression which amount to 8. For these two reasons, a better understanding of the non-factorisable corrections to single-top production at LHC would be beneficial.

Slide six

In this work, we need to compute the non-factorisable correction to the single-top production cross section. The different part of the cross-section are build from the double-real cross section made from ampltiudes with two emitted gluons presented to the top left of the slide, the real-virtual cross section made from amplitude with one emitted gluon, presented to the bottom left of the slide, and the double vitual cross section, build from the one- and two-loop amplitudes, presented to the right of the slide. We want to keep an exact dependence on the different kinematic invariants and the two masses, m_t and m_W. The master two-loop integrals are evaluated numerically with the auxiliary mass flow method. I refer you to the talk Christian gave yesterday in the case of W/Z boson pair production for more details about this numerical method.

Slide seven

The non-factorisable QCD corrections have some peculiar propreties. All non-abelian contributions including the 3-gluons vertex diagrams vanish. As an example, I consider the two-loop amplitude. It needs to be projected on the Born amplitude to contribute to the NNLO. This projection makes the the three-gluons vertex diagrams vanish since it is proportional to the trace of a color generator. Therefore, upon interference, the non-abelian part of the amplitude disappears and the amplitude is, effectively, Abelian.

Slide eight

Amplitudes display divergences. However, for non-factorisable contribution, they are only of infrared origin. Indeed, renormalised diagrams are absent due to colour conservation. These two renormalised diagrams, once correctly interfered, are proportional to the trace of a color generator. In addition, the infrared divergences can only be of soft origin. It can be understand qualitatively through the following argument: the collinear singularities in physical gauge can occur when an emitted gluon is absorbed by the same leg. This kind of contribution belongs to factorisable corrections.

Slide nine

I want now to focus on the obtention of the two-loop amplitude. First, we perform the standard procedure by performing an analytic reduction with KIRA and FireFly keeping full dependences on the four scale: the top and W masses, and the mandelstam s and t. The analytic reduction took up to 4 days on 20 cores. We end up with 428 two-loop master integrals splitted into 18 families.

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The master evaluation is done numerically. We use a method based on the auxiliary mass flow method. We add an imaginary part to the W boson mass. We work in the lower complex plane to ensure causality. I change variable by setting $x = -i\eta$. It is now possible to derive differential equation where I is the set of the 428 master integrals. Then, the idea of the method can be sum up in two sentences: Stepping from the boundary at $x = -i\infty$, via regular points to the physical mass at $x = 0$. Step size is limited by singularities of the equation.
Let me explain this in more details.

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First of all, we start from the boundary at $y = 1/x = 0$. This is a singular point. We can step into the complex plane by setting an ansatz for I which is a power-log expansion. Then, we are in the lower complex plane. We want to evaluate our set of master integral atthe regular point $x = 0$. As we expand around regular point, we provide an ansatz which is a simple Taylor expansion in x to an arbitrary order N. Our stepsize is limited by the presence of singularities which I indicate here in red. For instance, let's say I am laying at this regular point in blue. I am able to anywhere within the radius define by the distance between this point and the next singularity. Et cetera up to the point in $x = 0$. The relative error we make on the final result is fully determined by the order of the expansion N and the ration between the stepsize and the radius to the next singularities. We can therefore evaluate our master integral to an arbitrary precision.

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At the boundary, the integrals are simpler since propagators with an infinite mass are shrink to a point. This reduce drastically the number and the complexity of the master integrals. Indeed, in our case, we are left with the set of 17 integrals presented on this slide. Some of them are known analytically and some where not available or not know to sufficient order in ϵ.

In the end, all 428 master integrals can be evaluated numerically to 20 digits in 30 minutes on a single core.

We are now able to evaluate our two-loop amplitude at any point of the phase space.

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In this table, I compare the value of the pole we calculated for the two-loop amplitude, presented in the first line, with the one predicted with Catani operator I1, in the second line. The accuracy of the first pole is of 15 digits. We are now able to evaluate the two-loop amplitude at any point of the phase space to any desired accuracy. We prepare a Vegas grid on the Born amplitude and then generate 10 sets of 10^5 points for which we evaluate the value of the two-loop amplitude. The 10 independent runs give us an estimation of the error we make on the double-virtual cross-section, which is about 2%.

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I can now present the results. We evaluate the total cross-section in the case of a proton-proton collision at 13 TeV and factorisation scale $\mu_F = m_t$. The cross-section display a trivial dependence on the renormalisation scale μ_R. At $\mu_R = m_t$, the correction is about 0.2%. However, since non-factorisable corrections appear for the first time, we do not have any indication of a good scale choice as they are independent of LO, NLO and NNLO factorisable corrections. At $\mu_R = 40\,\mathrm{GeV}$, which is the typical transverse momentum of the top quark in this process, the correction becomes close to 0.35%. In comparison, the NNLO factorisable corrections to the NLO cross section is about 0.7%.

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We also computed some distribution of observable. I start by presenting the transverse moementum of the top quark. We fix the factorisation and the renormalisation scale to the same value. The red line is evaluated at $\mu = m_t$ and we vary it by a factor 2. The green line corresponds to the scale $\mu = 40\,\mathrm{GeV}$. We see a significant dependence of the correction with respect the $p_\perp^t$ value. At small pt, the correction is negative. This behaviour is the same as the one for the double-virtual corrections. It is interesting to note that the corrections vanish at $50\,\mathrm{GeV}$ for any scale choice. The factorisable corrections vanish at $30\,\mathrm{GeV}$. Therefore, in some part of the phase space at low $p_{\perp,top}$, where the peak of the distribution is, non-factorisable corrections are dominant.

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I come to my conclusion. In this work, we have computed the missing part of the NNLO QCD corrections to the t-channel single-top production: the non-factorisable corrections. We use the auxiliary mass flow method to evaluate the two-loop integrals. We proved that it was sufficiently robust to produce results relevant for phenomenology. We also presented the calcualtion of the one-loop, five-point amplitude which, due to the multiple mass scales, turns out to be non-trivial. In the end, it appears that non-factorisable corrections are smaller than, but quite comparable to, the factorisable one. Indeed, if a percent-level precision in single-top sutides can be reached, the non-factorisable effect will have to be taken into account.