Karlsruher Institut für Technologie

TRR 257 Particle Physics Phenomenology after the Higgs Discovery

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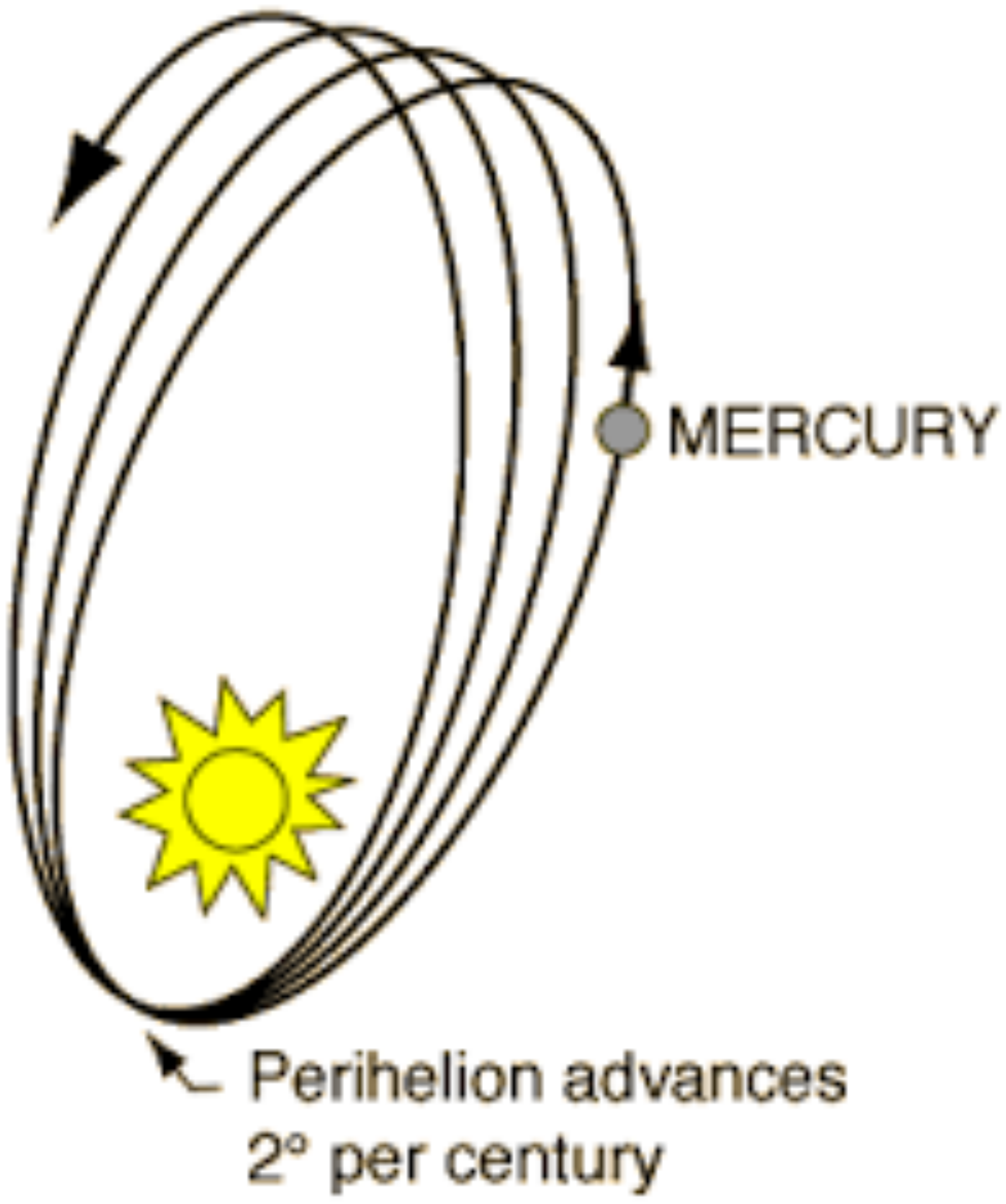
HIGH PRECISION PHYSICS AT THE LHC

Lake Louise Winter Institute, February 2019

PRECISION PHYSICS

Precision physics has an impressive track-record. Indeed, decisive indications of the correctness of general relativity and the relevance of quantum electrodynamics came from precision measurements.



Proof of general relativity: a 1% discrepancy with the Newtonian predictions

THE

PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 72, NO. 12 [illegible] 15, 1947

1256 LETTERS TO THE EDITOR

Precision Measurement of the Ratio of the Atomic 'g Values' in the $^2P_{3/2}$ and $^2P_{1/2}$ States of Gallium*

P. KUSCH AND H. M. FOLEY
Columbia University, New York, New York
November 3, 1947

THE measurement of the frequencies associated with the Zeeman splittings of the energy levels in [illegible] different atomic states in a constant magnetic field per[illegible] a determination of the ratio of the g_J values of the [illegible] states. This determination involves only an accur[illegible]urement of the frequencies, and does not requ[illegible] edge of the value of the constant magnetic [illegible]

Using the atomic beam magnetic res[illegible] we have measured six lines in the Zee[illegible] the $^2P_{3/2}$ state, and five lines of the ^2P[illegible] at a field strength of 380 gauss. The [illegible] course, complicated by the level splitti[illegible] the nuclear magnetic moments and ele[illegible] moments. At the field strength employ[illegible] the nuclear energy level pattern [illegible] Paschen-Back character.

The procedure [illegible] observations was to make a series [illegible] rements of the frequencies of the lines [illegible] $^2P_{1/2}$ states. In this way the effect of [illegible] field was minimized.

Beck[illegible] recently determined with high pre[illegible] magnetic moment and electric quadru[illegible] coefficients as well as the nuclear g [illegible] Ga[illegible] in both states. Their determinations [illegible] of any knowledge of g_J in either state.

Their results [illegible] the less accurate values obtained by [illegible] zero moment deflection method.

With [illegible] by Becker and Kusch and with [illegible] for g_J it is possible to determine the [illegible] the observed frequencies. The field [illegible] from either of the groups of lines in the [illegible] states are constant within the experi[illegible]tainty. What is more, there is excellent con[illegible] the results obtained from the data on the two [illegible] The mean value of magnetic field determined [illegible] the $^2P_{3/2}$ lines is less than the mean field value for the [illegible] lines by 0.65±0.02 gauss when the g_J values for the $^2P_{3/2}$ and $^2P_{1/2}$ states are taken to be 4/3 and 2/3, respectively. To remove this discrepancy we must assume for the ratio of the g_J values

$$\frac{g_{3/2}}{g_{1/2}} = 2.00344 \pm 0.00012 = 2 + \Delta.$$

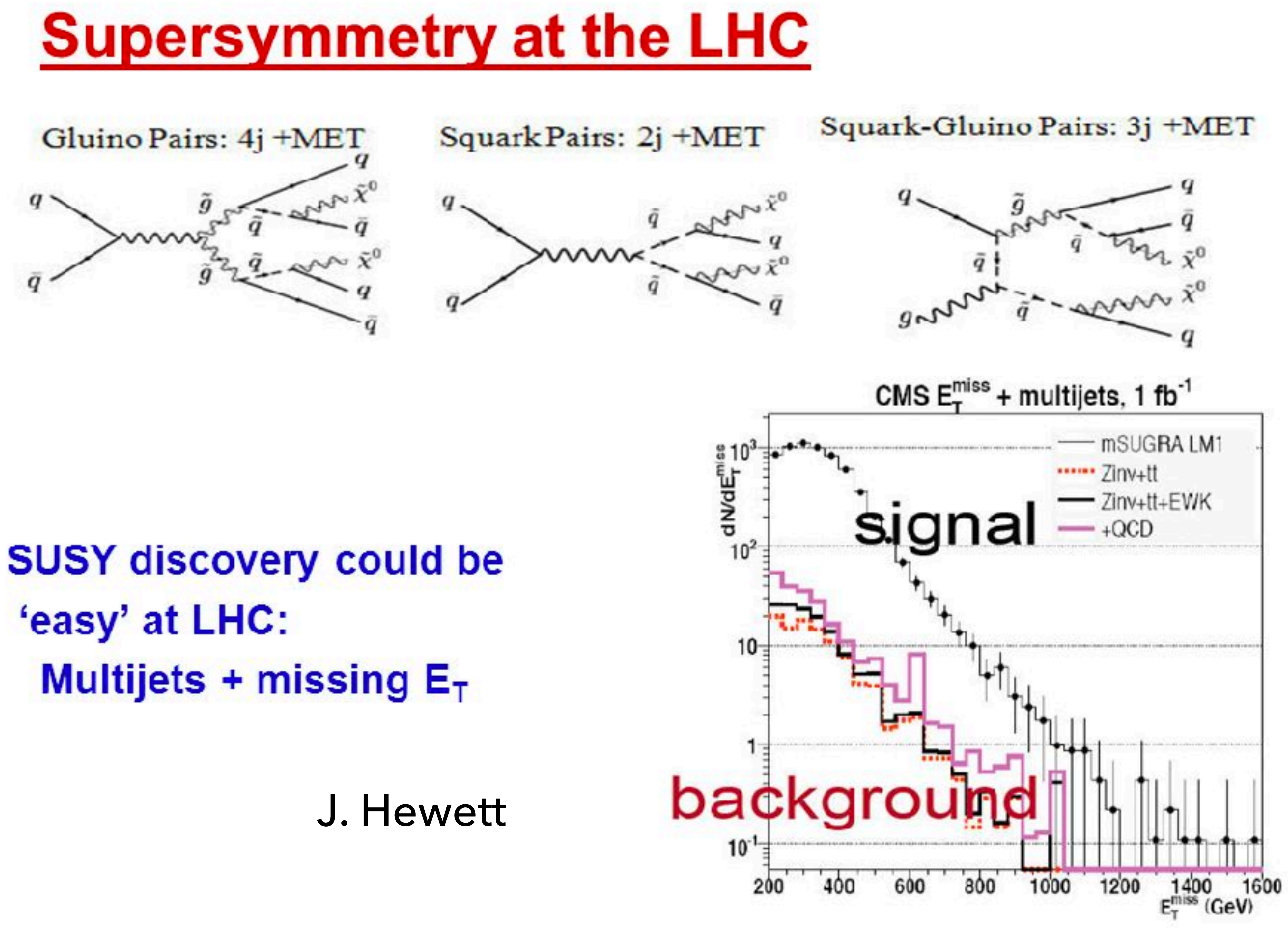
If the electronic configuration in these states is accurately described by Russell-Saunders coupling the above discrepancy must be assigned to a change in the g value of the intrinsic moment of the electron or of the orbital moment from their accepted values. If the electron spin g value $g_S = 2 + \delta_S$ and the orbital g value $g_L = 1 + \delta_L$, then $\Delta = \frac{2}{3}(\delta_S - 2\delta_L)$. Our present experiments, even assuming Russell-Saunders coupling, do not permit any evaluation of δ_S and δ_L. However, the discrepancy could be accounted for by taking $g_S = 2.00229 \pm 0.00008$ and $g_L = 1$, or alternatively $g_S = 2$ and $g_L = 0.99886 \pm 0.00004$.

The experiments reported in this note are of a preliminary nature. It is our intention to continue these studies with other atomic systems to clarify the role of the coupling in this phenomenon.

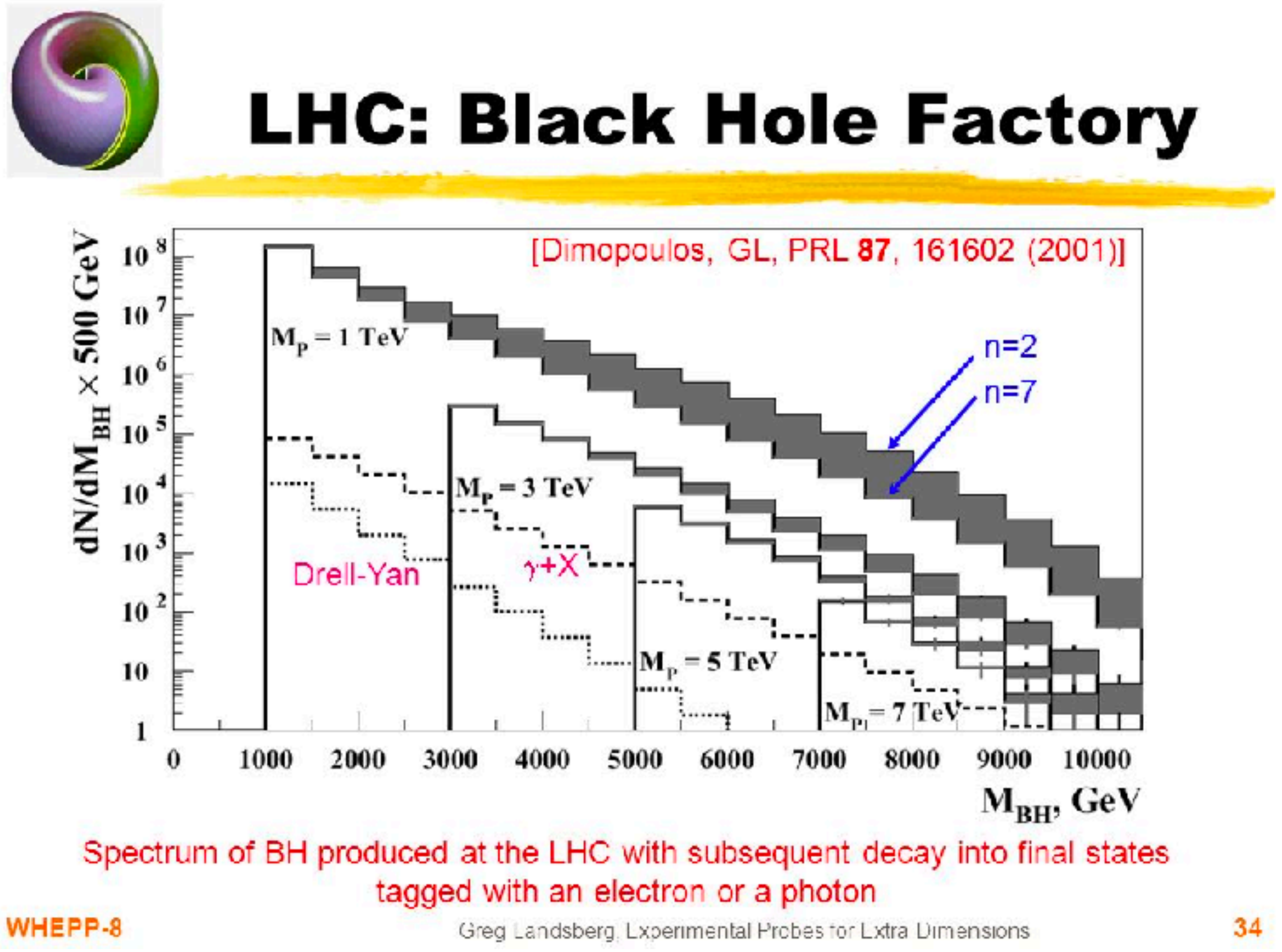
The authors are grateful to Professors G. Breit and I. I. Rabi for suggestions which stimulated these experiments.

* Publication assisted by the Ernest Kempton Adams Fund for Physical Research of Columbia University.
[1] To be published.
[2] N. A. Renzetti, Phys. Rev. 57, 753 (1940).

Discovery of Quantum Electrodynamics: 0.1% discrepancy with predictions of Quantum Mechanics



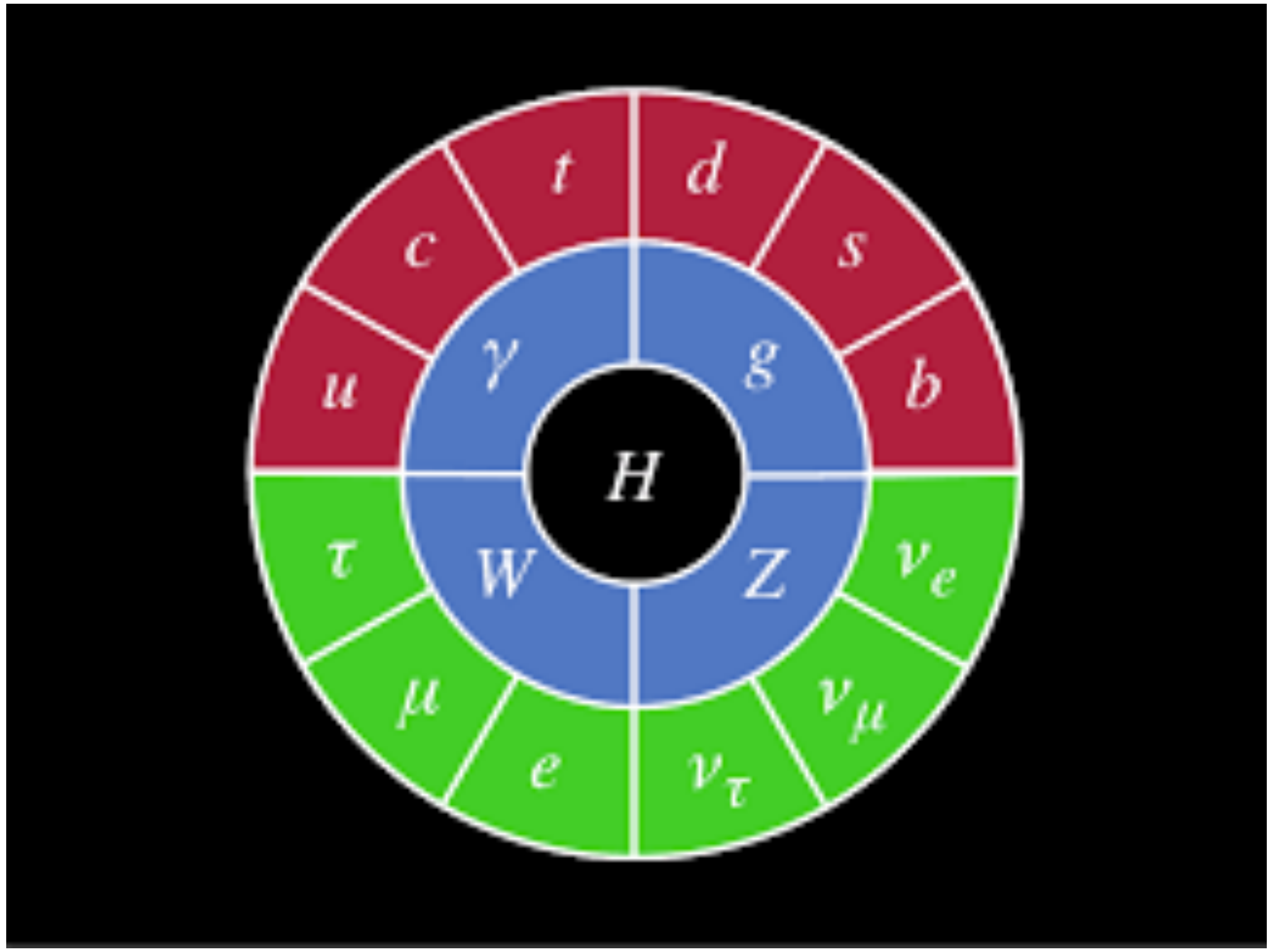
J. Hewett

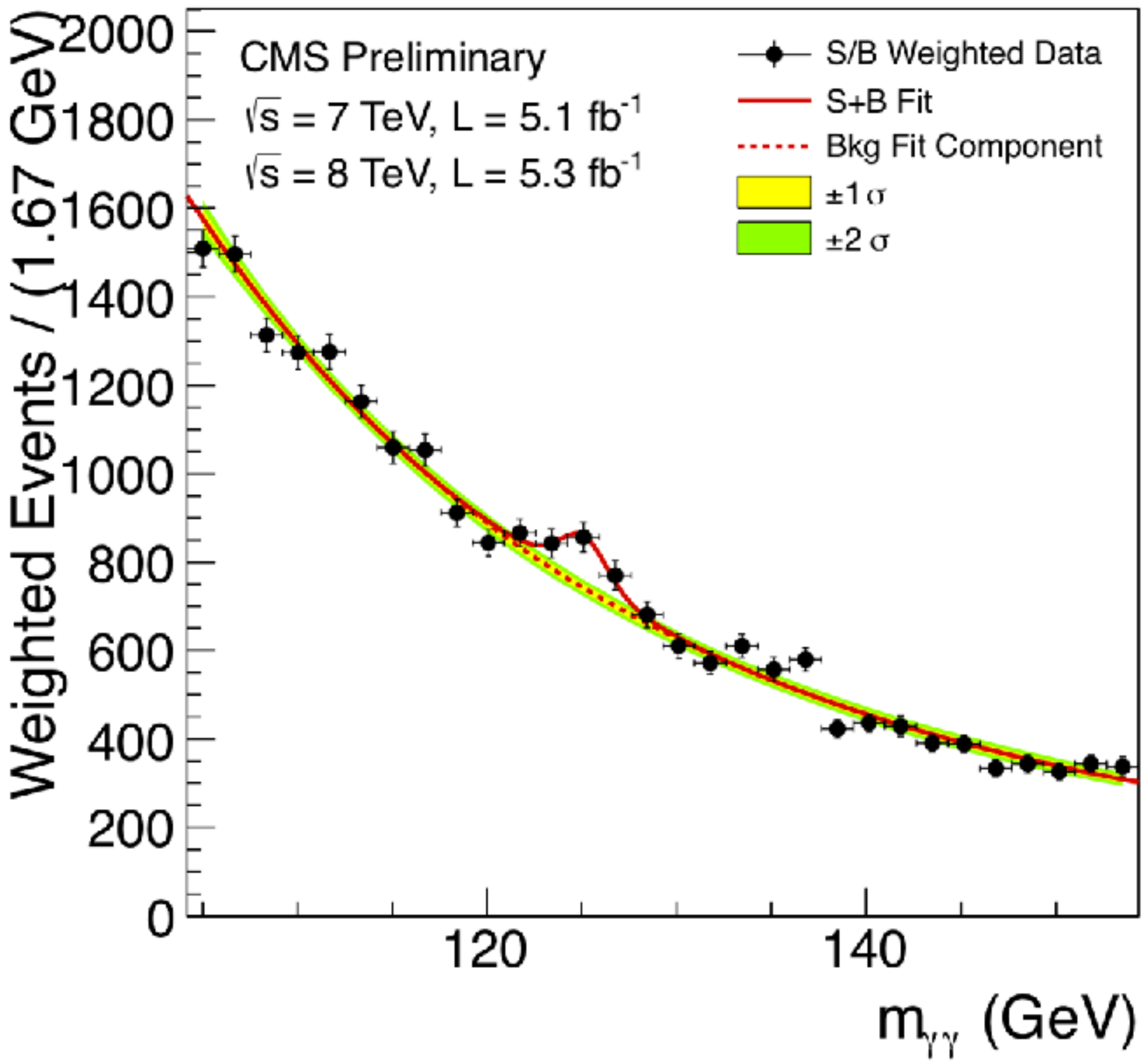


G. Landsberg

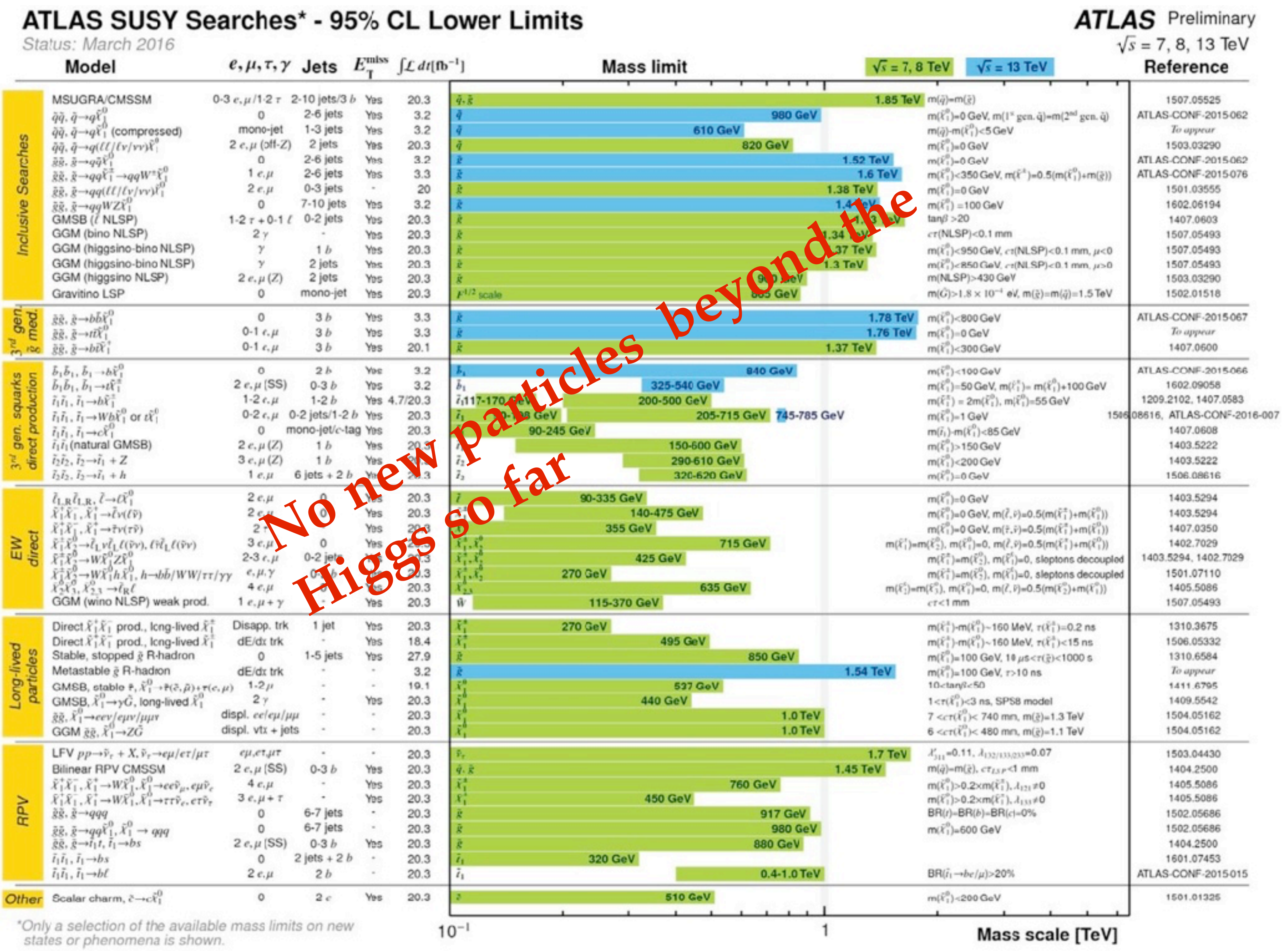
The discovery of low-scale BSM physics at the LHC was expected to be easy. Strong and unambiguous signals were expected. Perturbative QCD and precise predictions for physical processes were only needed to ensure that nothing unusual occurs with the backgrounds, and to clarify the nature of BSM signals after their discovery.

A Higgs boson — a particle that is supposed to hold the Standard Model together — was discovered six years ago. Although this discovery is of a fundamental importance, Higgs boson was viewed as an ``inevitable'' part of the Standard Model.





THE REALITY



No indication of any O(TeV)-scale physics beyond the Standard Model. Direct production is running out of steam.

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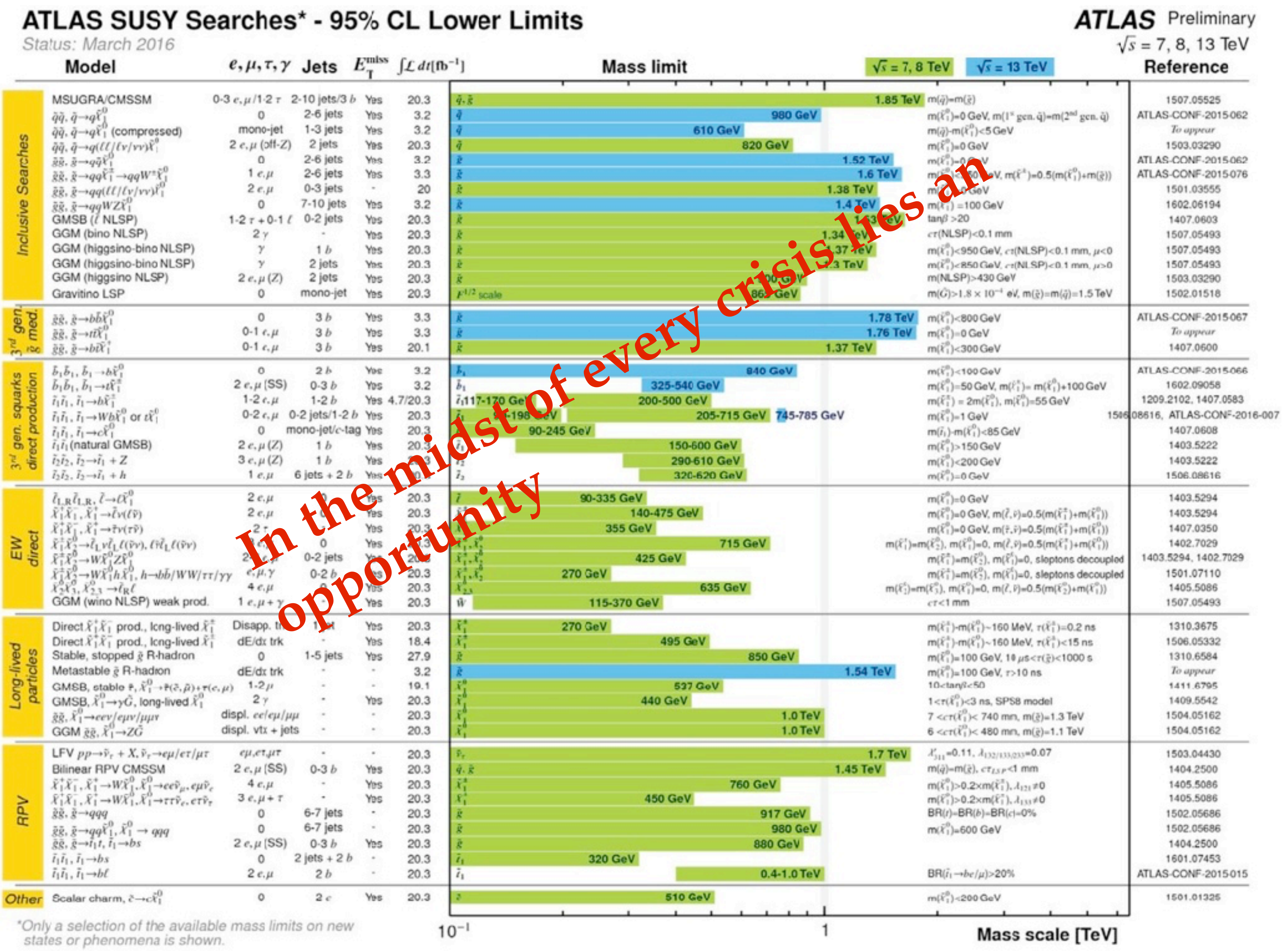


No indication of any O(TeV) physics beyond the Standard Model. Direct production is running out of steam.

We should turn the LHC into a precision machine and try to discover or constrain physics beyond the Standard Model indirectly, by confronting results of precise experimental measurements of many different SM processes with equally precise theoretical predictions.

NEW FOCUS OF PRECISION PHYSICS

$$N_{\text{events}} = N_{\text{events}}^{\text{SM}} + ?$$

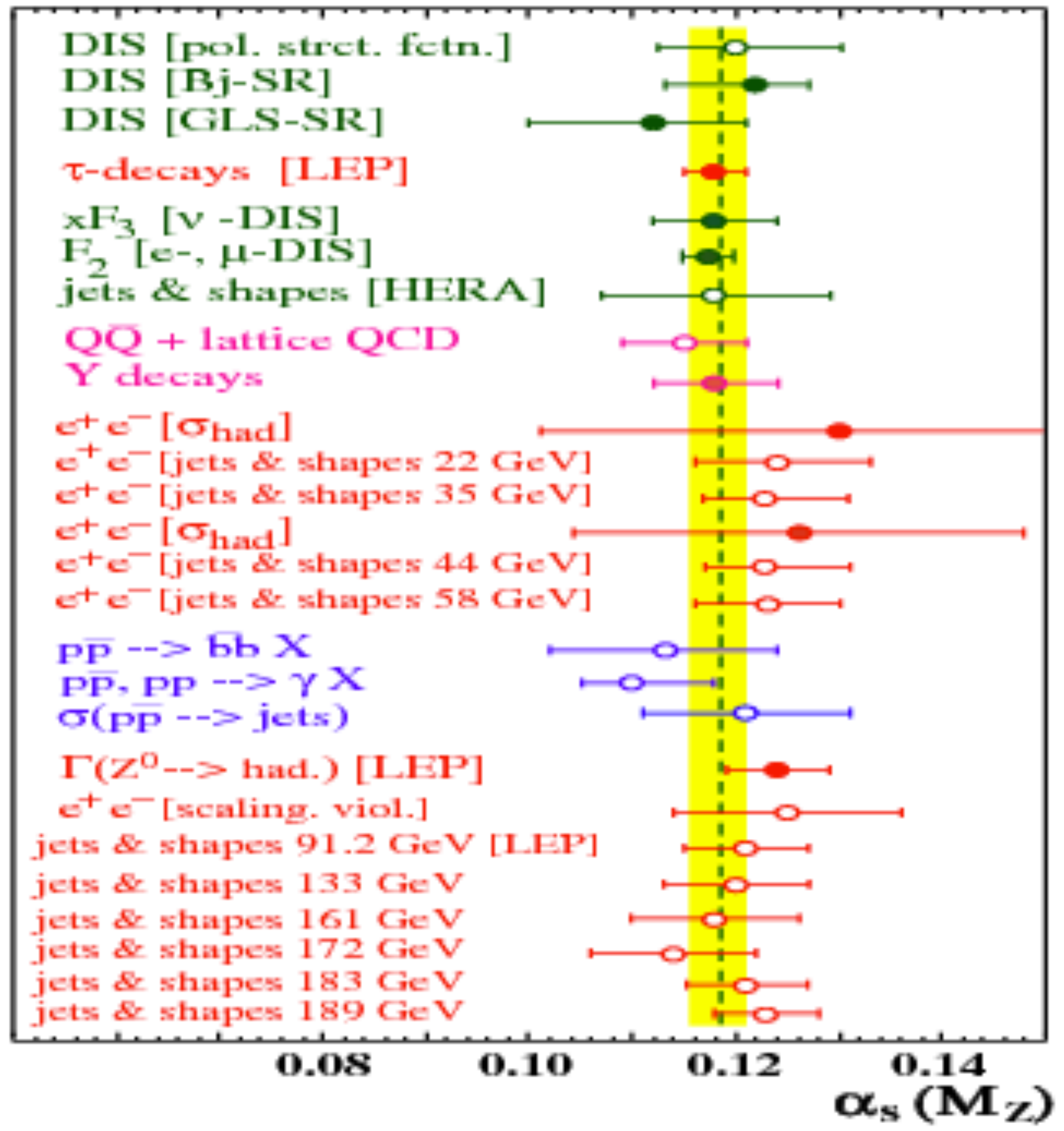
The LHC was not envisaged as a precision machine, but it can be turned into one, provided that QCD theory can keep up.

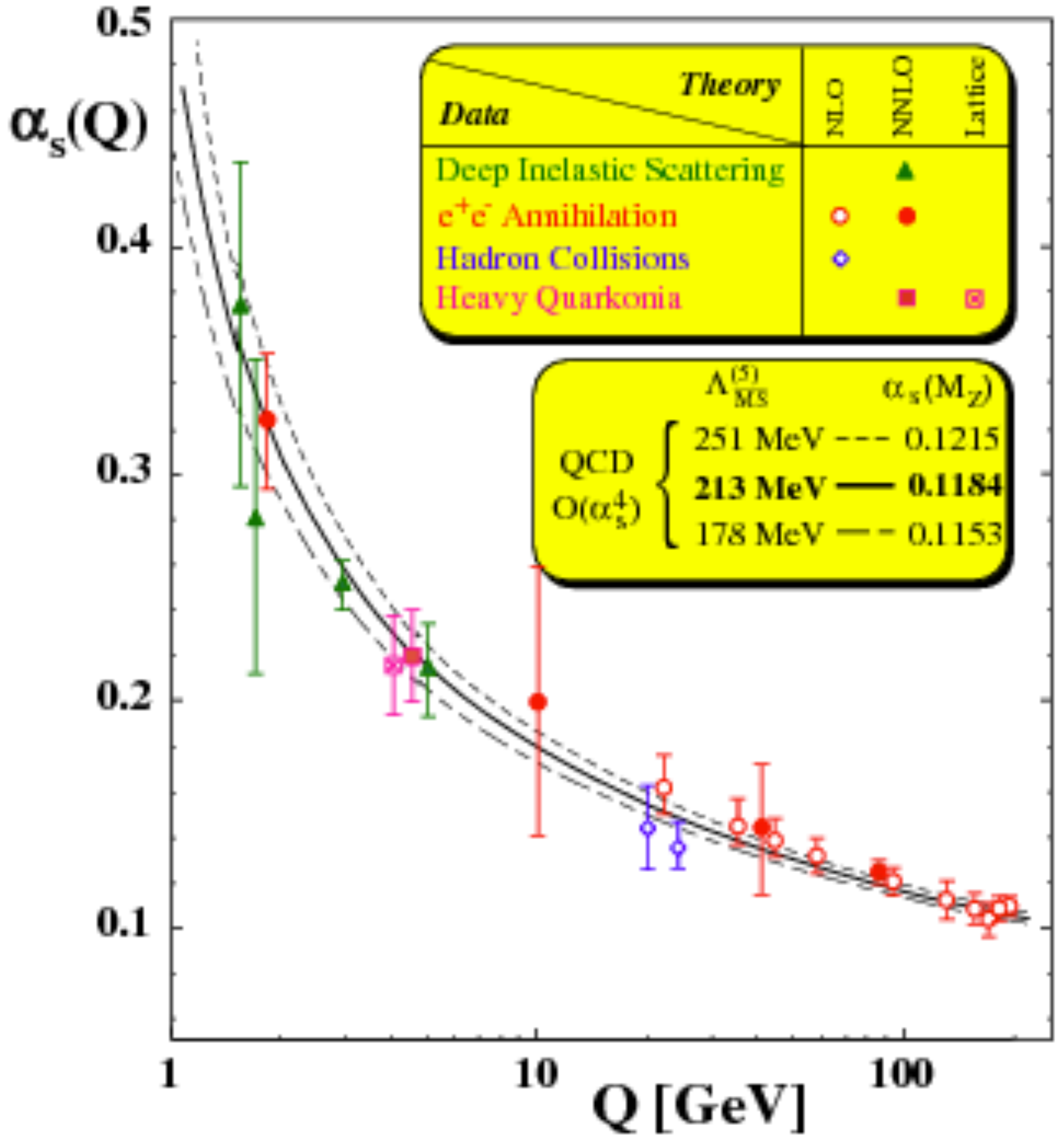
QCD LAGRANGIAN

How do we describe collisions of protons starting from a Lagrangian of quarks and gluons? We can do that because at short-distances perturbative description of strong interactions becomes possible, thanks to the phenomenon of asymptotic freedom.

$$\mathcal{L}_{\rm QCD} = \sum \bar{q}_j \left(i\hat{D} - m_j \right) q_j - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$

$$D_\mu = \partial_\mu - i g_s T^a A^a_\mu, \quad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + i g_s f^{abc} A^b_\mu A^c_\nu. \qquad [T^a, T^b] = i f_{abc} T^c$$





$$\alpha_s(\mu) = \frac{1}{\beta_0 \log \frac{\mu^2}{\Lambda^2}},$$

$$\beta_0 = \frac{33 - 2n_f}{12\pi} \sim 0.5|_{n_f=5}$$

$$\Lambda \approx 300 \text{ MeV}$$

COLLINEAR FACTORIZATION

Collinear factorization provides a well-established framework for the description of hard hadron collisions

$$d\sigma = \int dx_1 dx_2\, f_i(x_1) f_j(x_2) d\sigma_{ij} \mathcal{F}_j \left(1 + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{Q}\right)\right)$$

Collins, Soper, Sterman

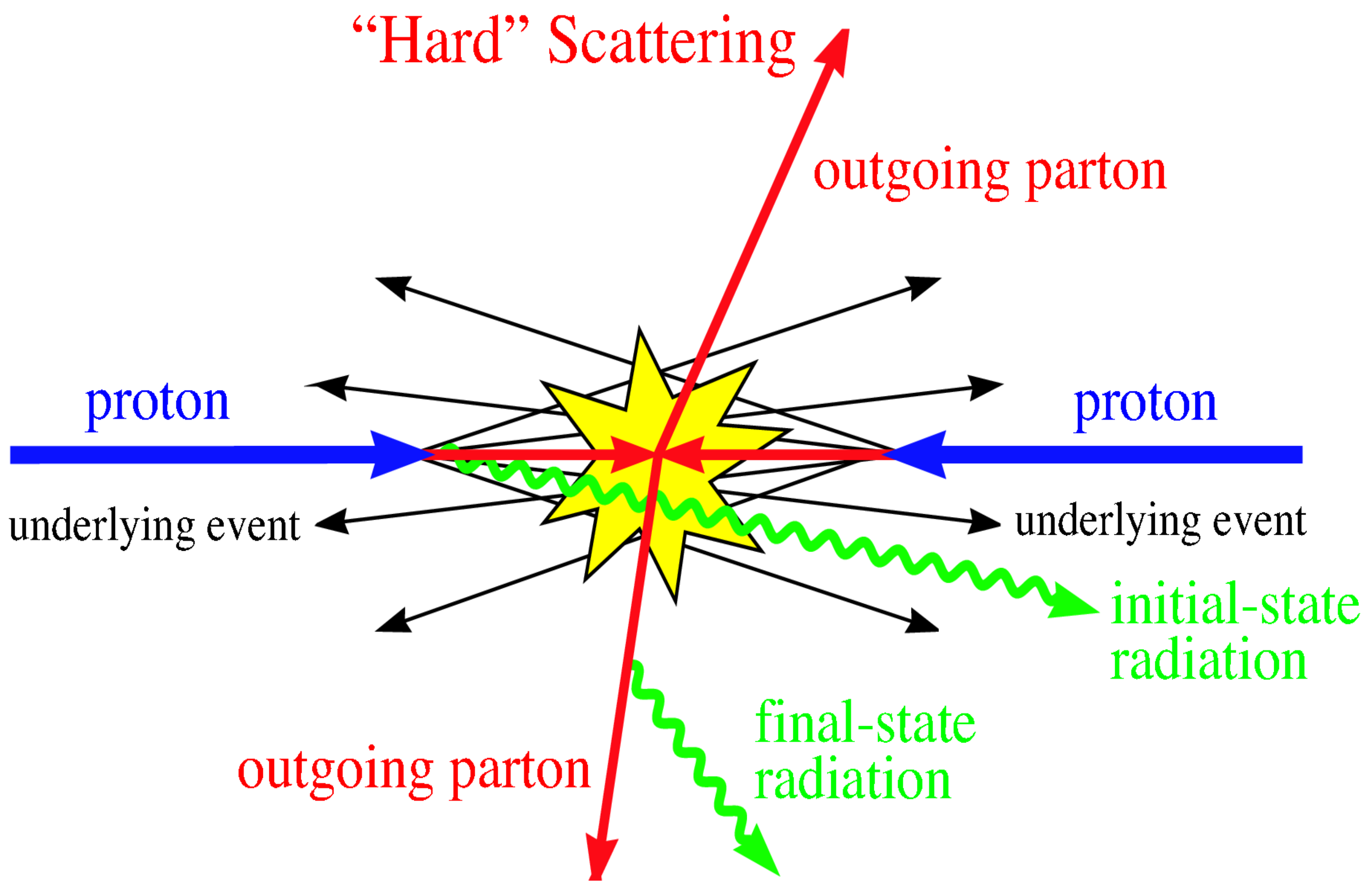
Except for parton distribution functions, all other non-perturbative effects are power-suppressed.

We can compute partonic cross sections in perturbation theory in the strong coupling constant (LO, NLO, NNLO, etc.).

$$d\sigma_{ij} = d\sigma_{ij,\rm LO} \left(1 + \alpha_s\, \Delta_{ij,\rm NLO} + \alpha_s^2\, \Delta_{ij,\rm NNLO} + ...\right)$$

Two or three orders in perturbative QCD can be studied without worrying about non-perturbative effects; however, it does not make sense to continue with even higher order pQCD computations without addressing generic non-perturbative corrections.

$$N_c \left(\frac{\alpha_s}{\pi}\right)^2 \sim \frac{\Lambda_{\rm QCD}}{Q}, \quad \Lambda_{\rm QCD} \sim 0.3\ {\rm GeV}, \quad Q \sim 30\ {\rm GeV}$$



THIS DISCUSSION IMPLIES THAT —AT LEAST ON THE THEORY SIDE — IT SHOULD BE POSSIBLE TO REACH A (FEW) PERCENT PRECISION FOR GENERIC LHC OBSERVABLES.

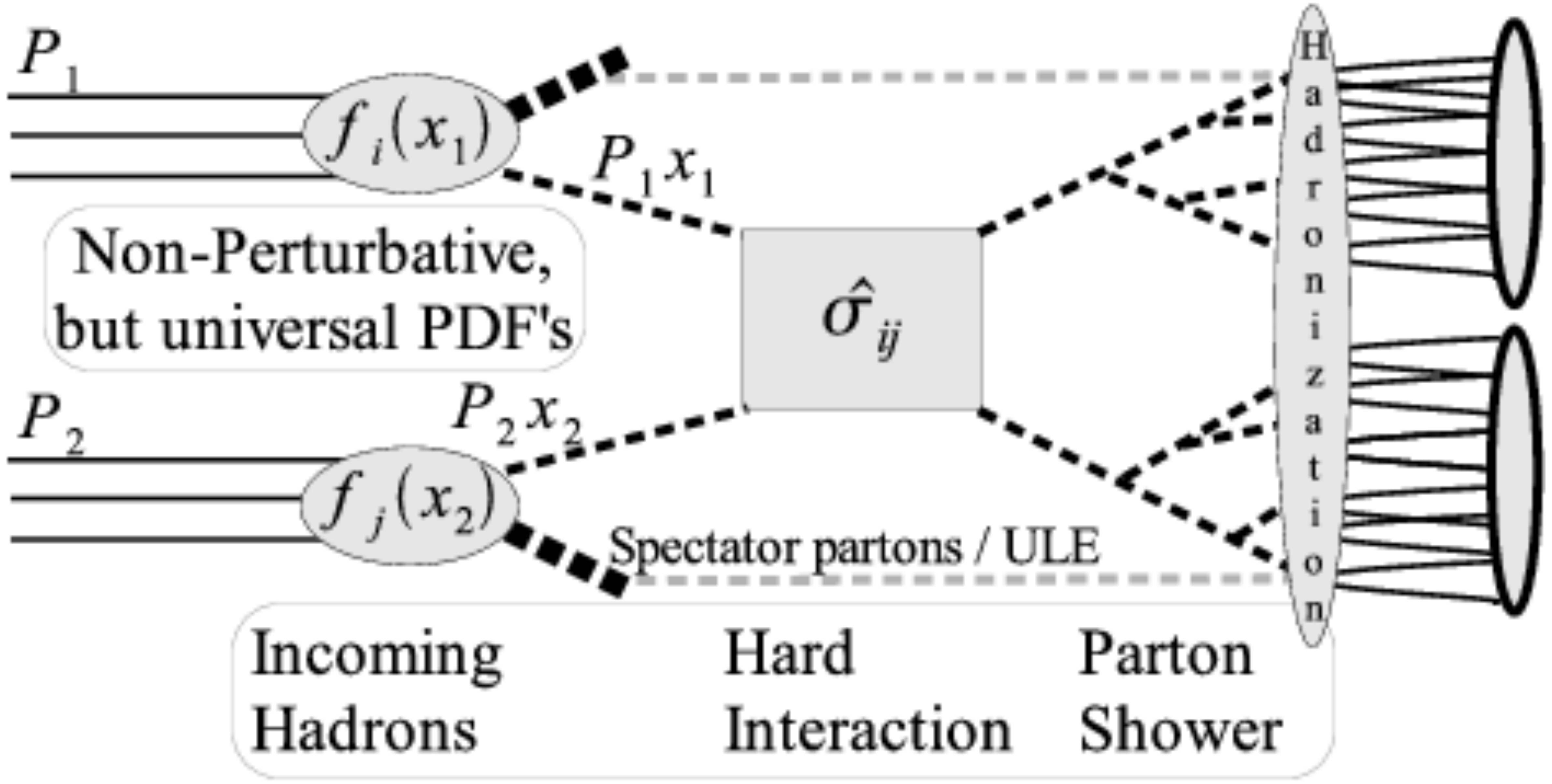
NON-PERTURBATIVE EFFECTS

Is the collinear factorization consistent with the known non-perturbative effects such as the underlying event, double parton scattering, hadronization, pile-up, color-reconnection etc. ?   Yes ! All these effects are collected in the power-suppressed correction!

Simulations of these non-perturbative effects involve models tuned to describe data. This is a serious obstacle for the precision physics program that forces us to choose observables in such a way that sensitivity to non-perturbative effects is avoided.

$$d\sigma = \int dx_1 dx_2\, f_i(x_1) f_j(x_2) d\sigma_{ij} \mathcal{F}_j \left(1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{Q}\right)\right)$$

Pile-up, hadronization, double-parton scattering, color-reconnection, underlying event etc.



THE TOP QUARK MASS — A COUNTER-EXAMPLE

Not everything that we want to know falls into a category of infra-red safe observables — a case in point is the mass of the top quark. Because of that determinations of the top quark mass at a hadron collider with very high precision are controversial. However, the top quark mass is important for clarifying very profound questions about our Universe so the quest continues.

Top quark is the only quark (and the only particle for that matter) that officially got THREE(!) different masses according to PDG !

$I(J^P) = 0(\frac{1}{2}^+)$

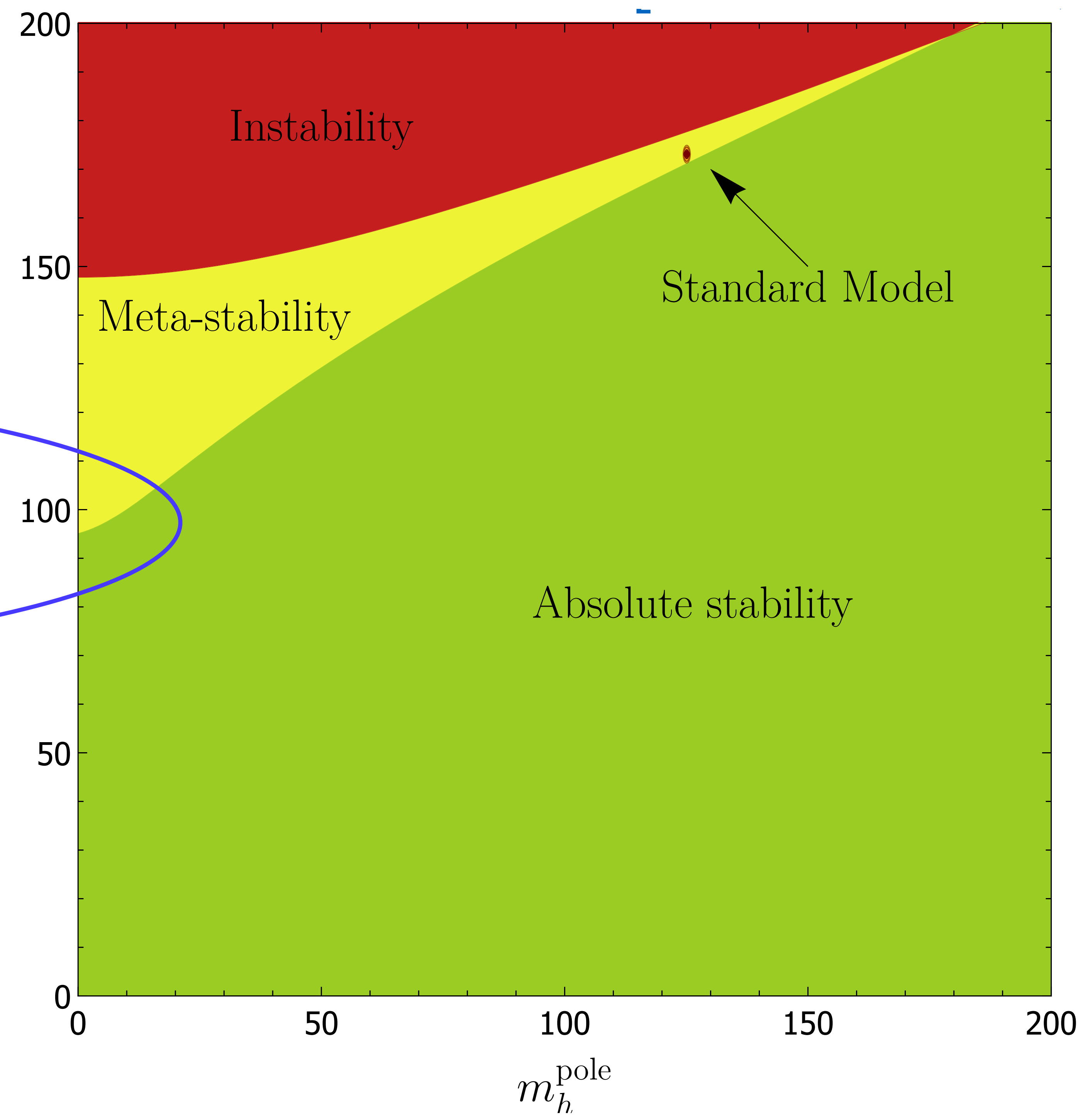
Charge = $\frac{2}{3}$ e Top = +1

Mass (direct measurements) $m = 173.1 \pm 0.6$ GeV [a,b] (S = 1.6)
Mass from cross-section measurements) $m = 160^{+5}_{-4}$ GeV [a]
Mass (Pole from cross-section measurements) $m = 173.5 \pm 1.1$ GeV
$m_t - m_{\bar{t}} = -0.2 \pm 0.5$ GeV (S = 1.1)
Full width $\Gamma = 1.41^{+0.19}_{-0.15}$ GeV (S = 1.4)
$\Gamma(W\,b)/\Gamma(W\,q(q = b,\,s,\,d)) = 0.957 \pm 0.034$ (S = 1.5)

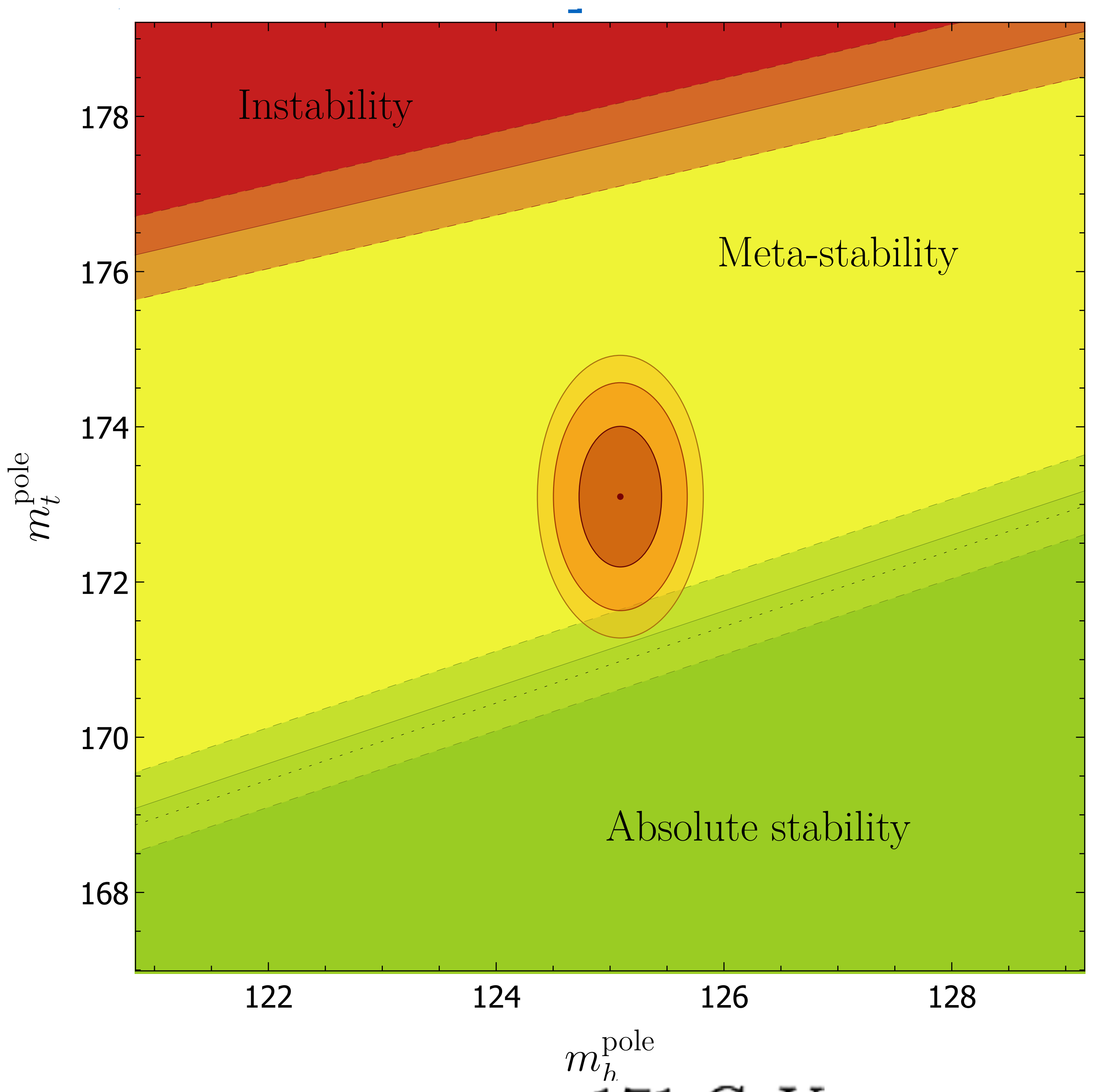
***t*-quark EW Couplings**

$F_0 = 0.685 \pm 0.020$
$F_- = 0.320 \pm 0.013$
$F_+ = 0.002 \pm 0.011$
$F_{V+A} < 0.29$, CL = 95%

$$m_{\text{obs}} = m_{\text{top}} + \mathcal{O}(\Lambda_{\text{QCD}})$$



$$\tau_{\text{SM}} \sim 10^{139^{102}_{-51}} \text{ years}$$



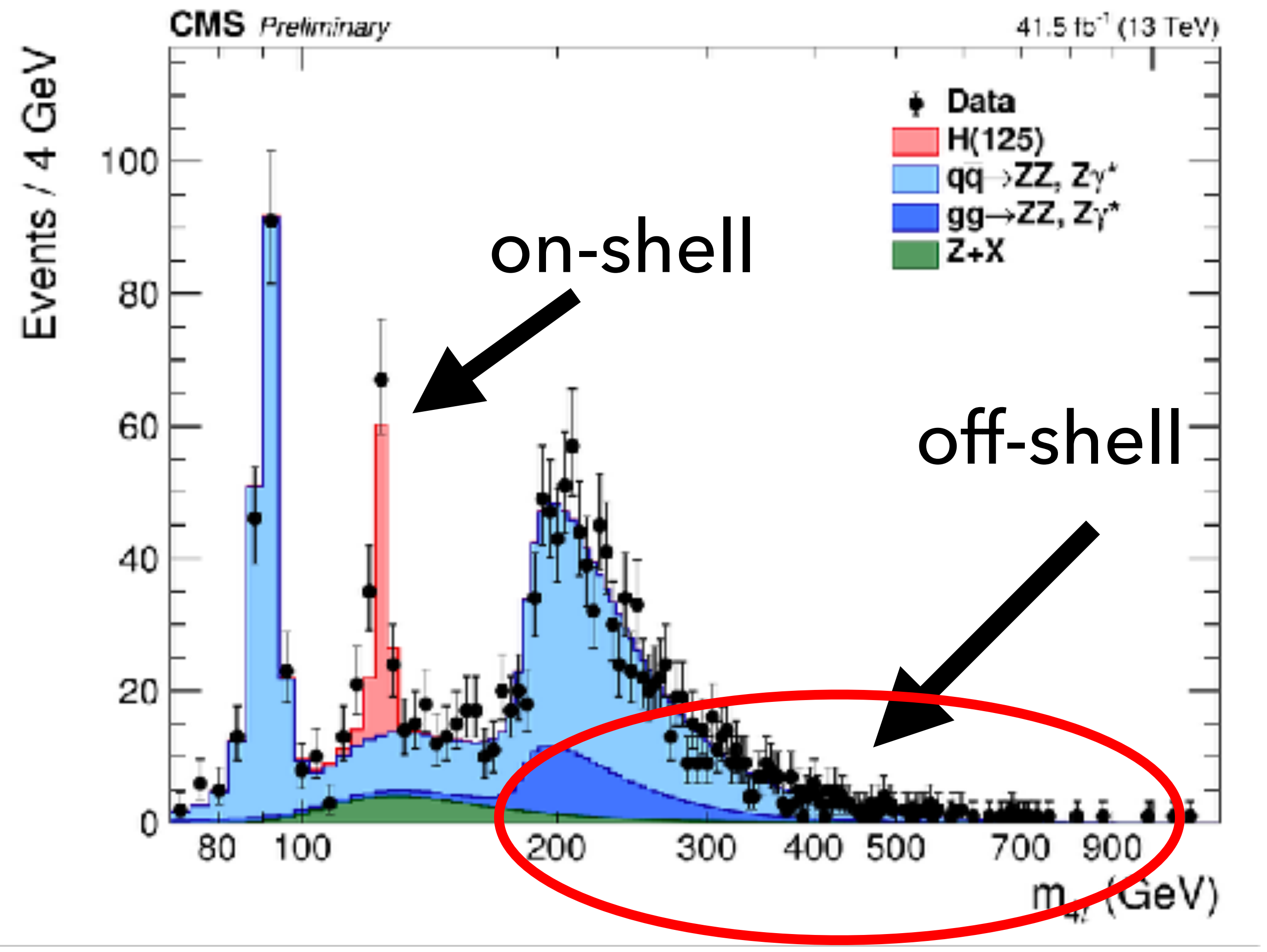
$m_t < 171$ GeV

is needed for the vacuum stability

Physics driven by precision

OFF-SHELL MEASUREMENTS: THE HIGGS WIDTH

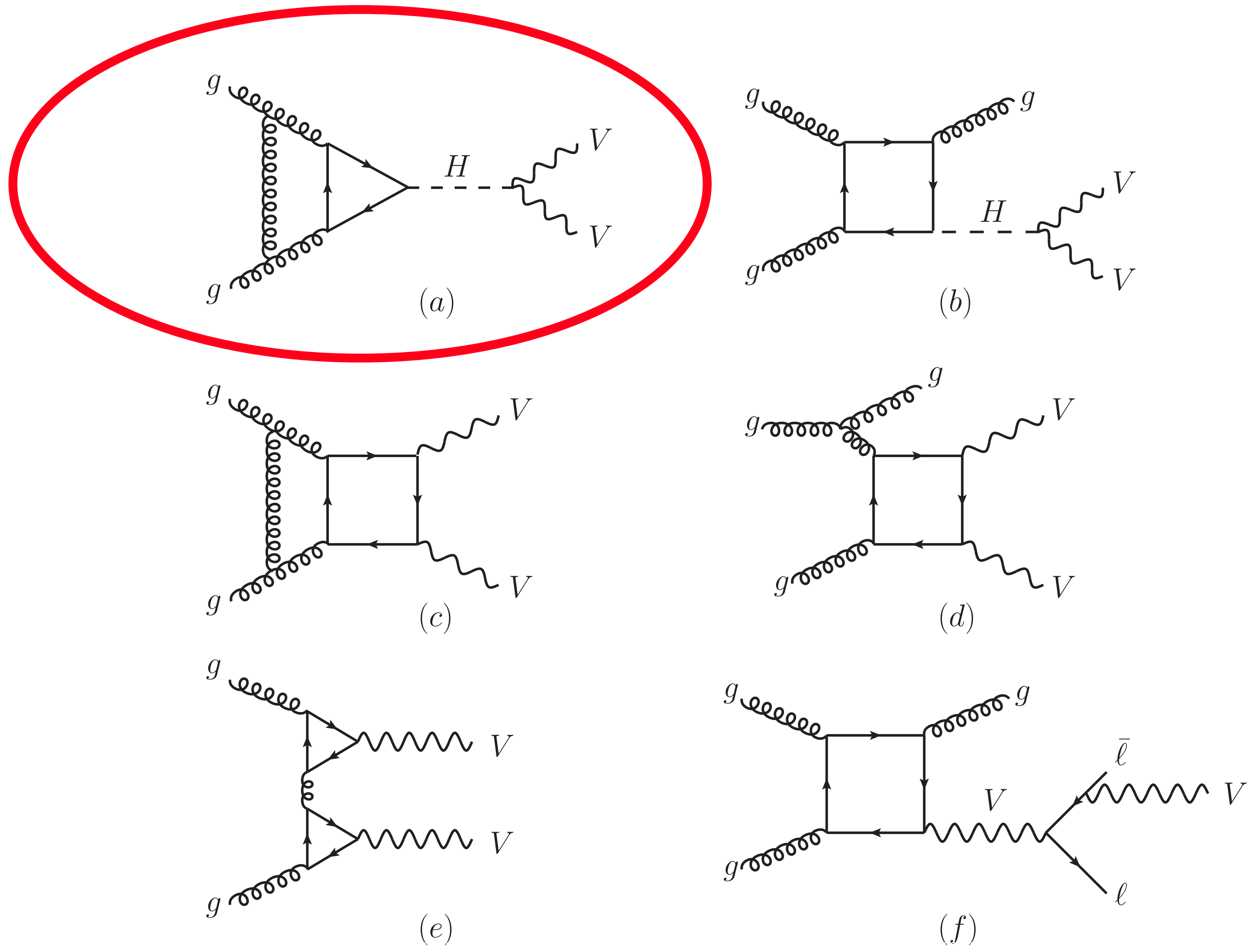
The Higgs boson width in the SM is 4 MeV. It would be interesting to measure it since it can affected by exotic Higgs decays. Impossible to do that at the mass peak; can try ``off-shell'' .



$$\sigma_{\rm on} \propto g_i^2 g_f^2/\Gamma_H \qquad \sigma_{\rm off} \propto g_i^2 g_f^2$$

$$\Rightarrow \Gamma_H \propto \frac{\sigma_{\rm off}}{\sigma_{\rm on}}$$ **indirect constraint on width**

Tiny signal/background ratio. Need precise prediction for ZZ production both in quark-antiquark and gluon fusion, including the interference with the off-shell Higgs in the gg channel.



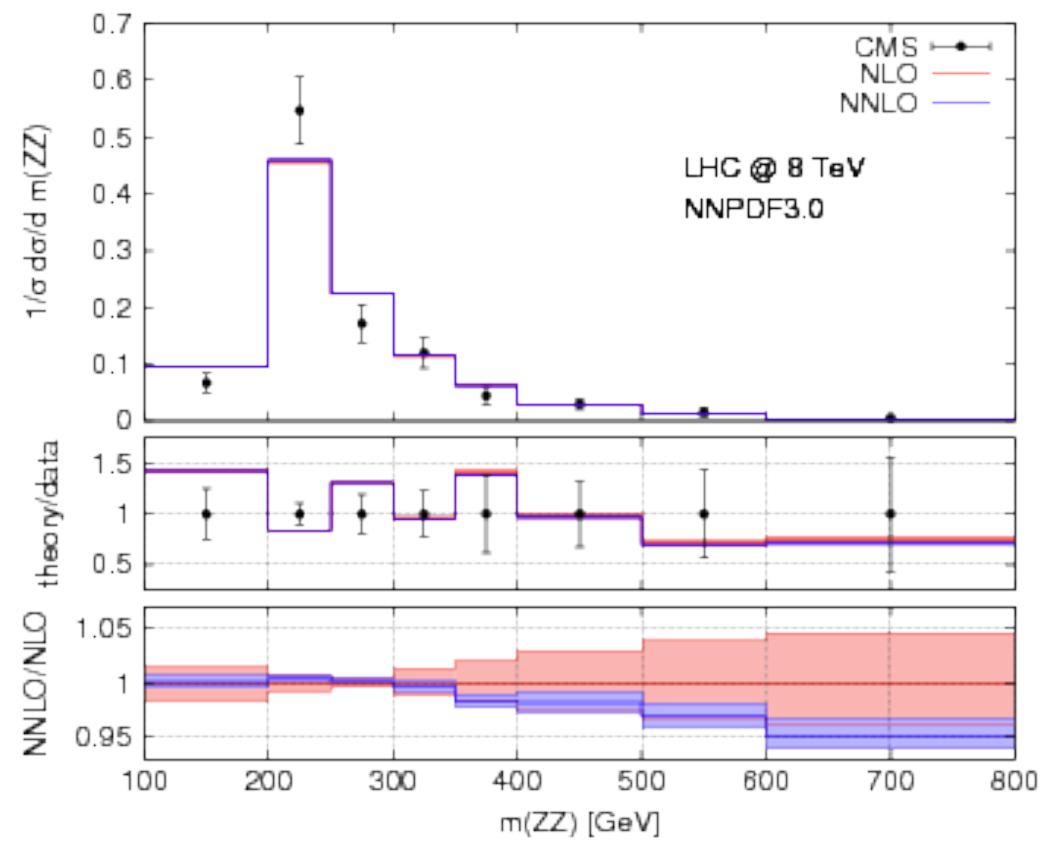
		4ℓ	$2\ell 2\nu$
(a)	total gg ($\Gamma_{\rm H} = \Gamma_{\rm H}^{\rm SM}$)	1.8 ±0.3	9.6 ±1.5
	gg signal component ($\Gamma_{\rm H} = \Gamma_{\rm H}^{\rm SM}$)	1.3 ±0.2	4.7 ±0.6
	gg background component	2.3 ±0.4	10.8 ±1.7
(b)	total gg ($\Gamma_{\rm H} = 10 \times \Gamma_{\rm H}^{\rm SM}$)	9.9 ±1.2	39.8 ±5.2
(c)	total VBF ($\Gamma_{\rm H} = \Gamma_{\rm H}^{\rm SM}$)	0.23 ±0.01	0.90 ±0.05
	VBF signal component ($\Gamma_{\rm H} = \Gamma_{\rm H}^{\rm SM}$)	0.11 ±0.01	0.32 ±0.02
	VBF background component	0.35±0.02	1.22 ±0.07
(d)	total VBF ($\Gamma_{\rm H} = 10 \times \Gamma_{\rm H}^{\rm SM}$)	0.77 ±0.04	2.40 ±0.14
(e)	$q\bar{q}$ background	9.3 ±0.7	47.6 ±4.0
(f)	other backgrounds	0.05 ±0.02	35.1 ±4.2
(a+c+e+f)	total expected ($\Gamma_{\rm H} = \Gamma_{\rm H}^{\rm SM}$)	11.4 ±0.8	93.2 ±6.0
(b+d+e+f)	total expected ($\Gamma_{\rm H} = 10 \times \Gamma_{\rm H}^{\rm SM}$)	20.1 ±1.4	124.9 ±7.8
	observed	11	91

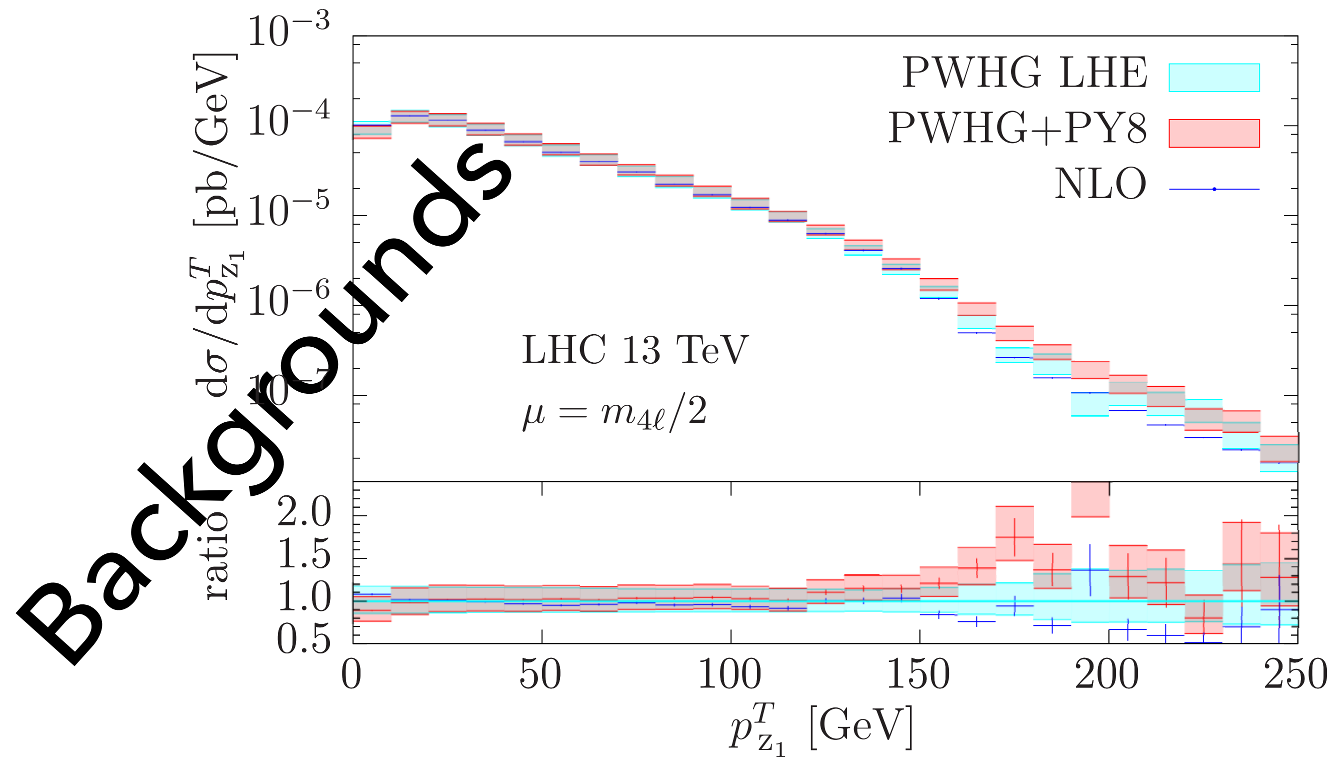
Table from one of the early CMS papers on the subject

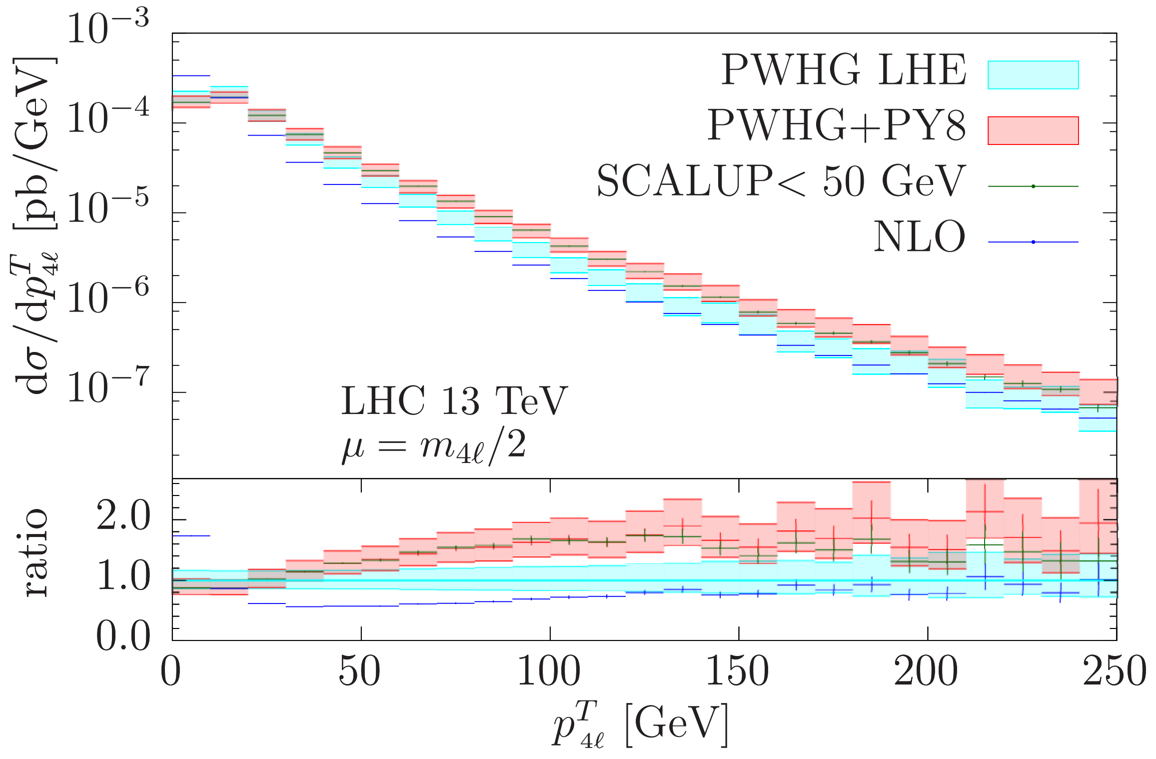
OFF-SHELL MEASUREMENTS: THE HIGGS WIDTH

Quark-antiquark annihilation to ZZ is known through NNLO QCD and gg -> ZZ through NLO QCD (two loops), including interference with the signal. Integrals with top quark loops are known approximately. Close proximity of K-factors for the signal and the background.

The CMS result: $\Gamma_H = 3.2^{+2.8}_{-2.2}$ MeV@68%CL $\qquad \Gamma_{\rm SM} \approx 4$ MeV

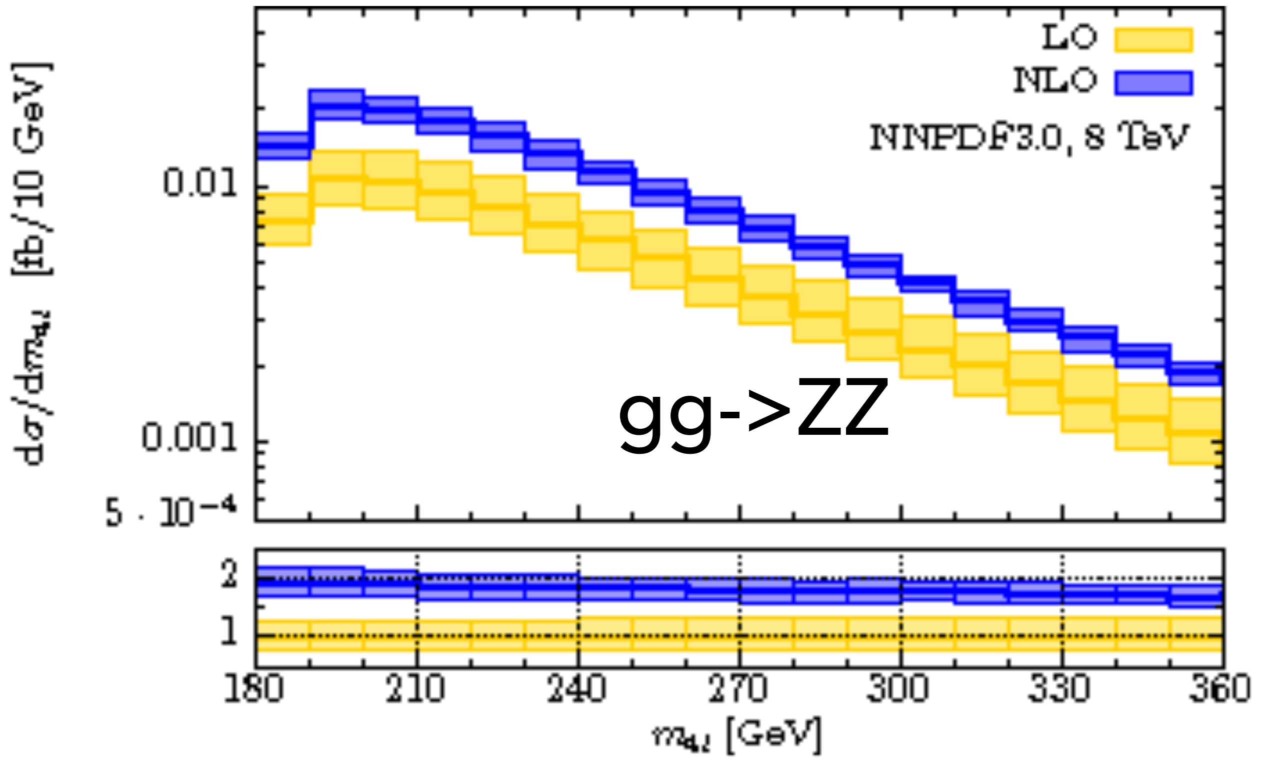


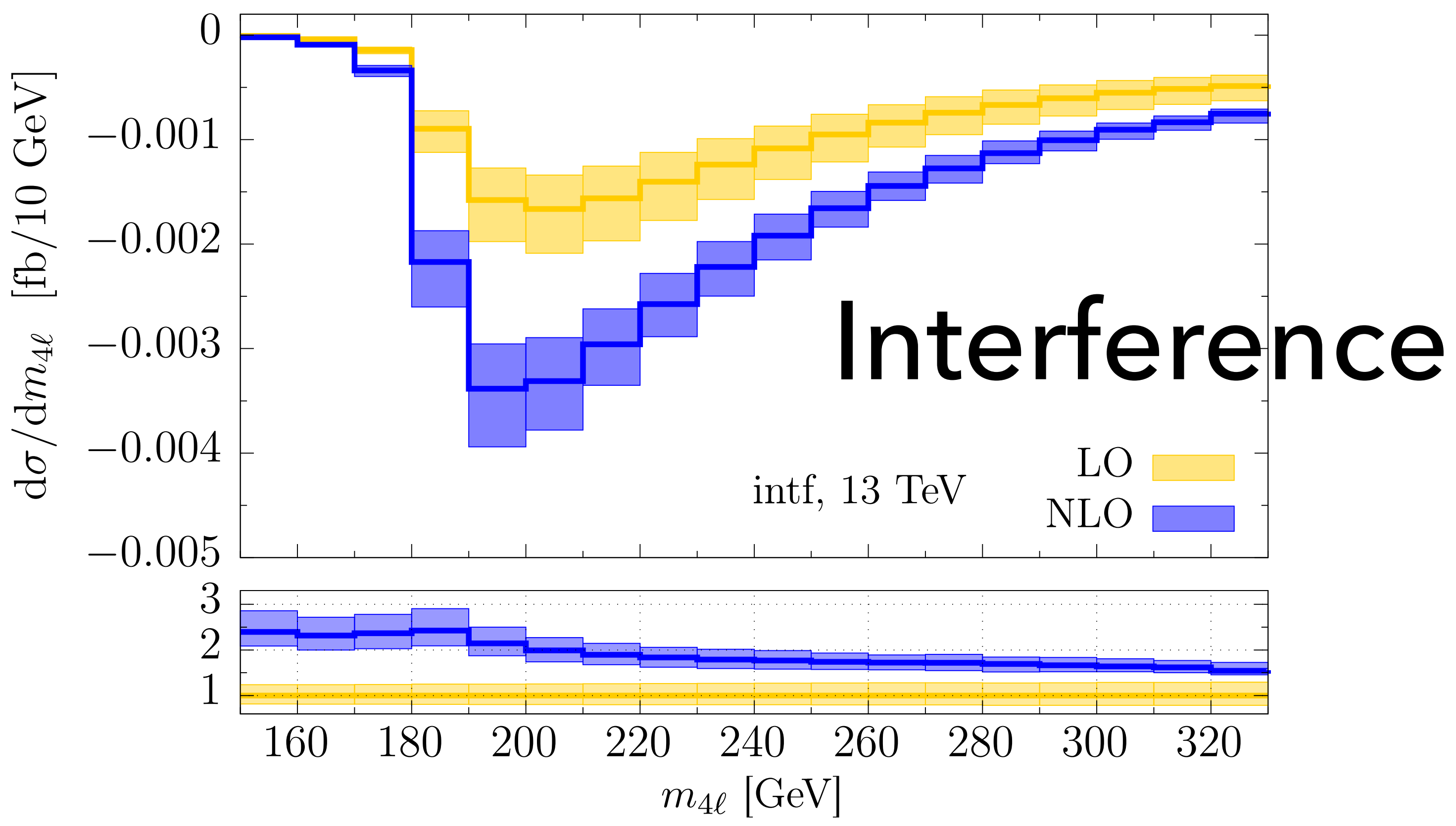




M. Grazzini, S. Kallweit, P. Maierhoefer, D. Rathlev

S. Alioloi, F. Caola, J. Luisoni, R. Rontsch.



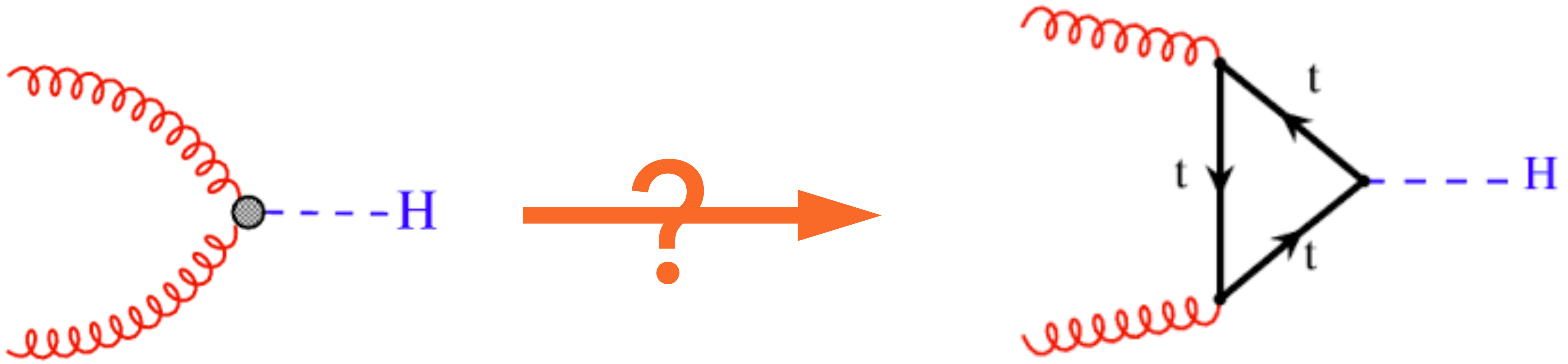


F. Caola, K. M., R. Rontsch, L. Tancredi; interference also: J. Campbell, M. Czakon, K. Ellis, S.Kirchner

HIGGS BOSON PRODUCTION WITH THE HIGHEST TRANSVERSE MOMENTUM

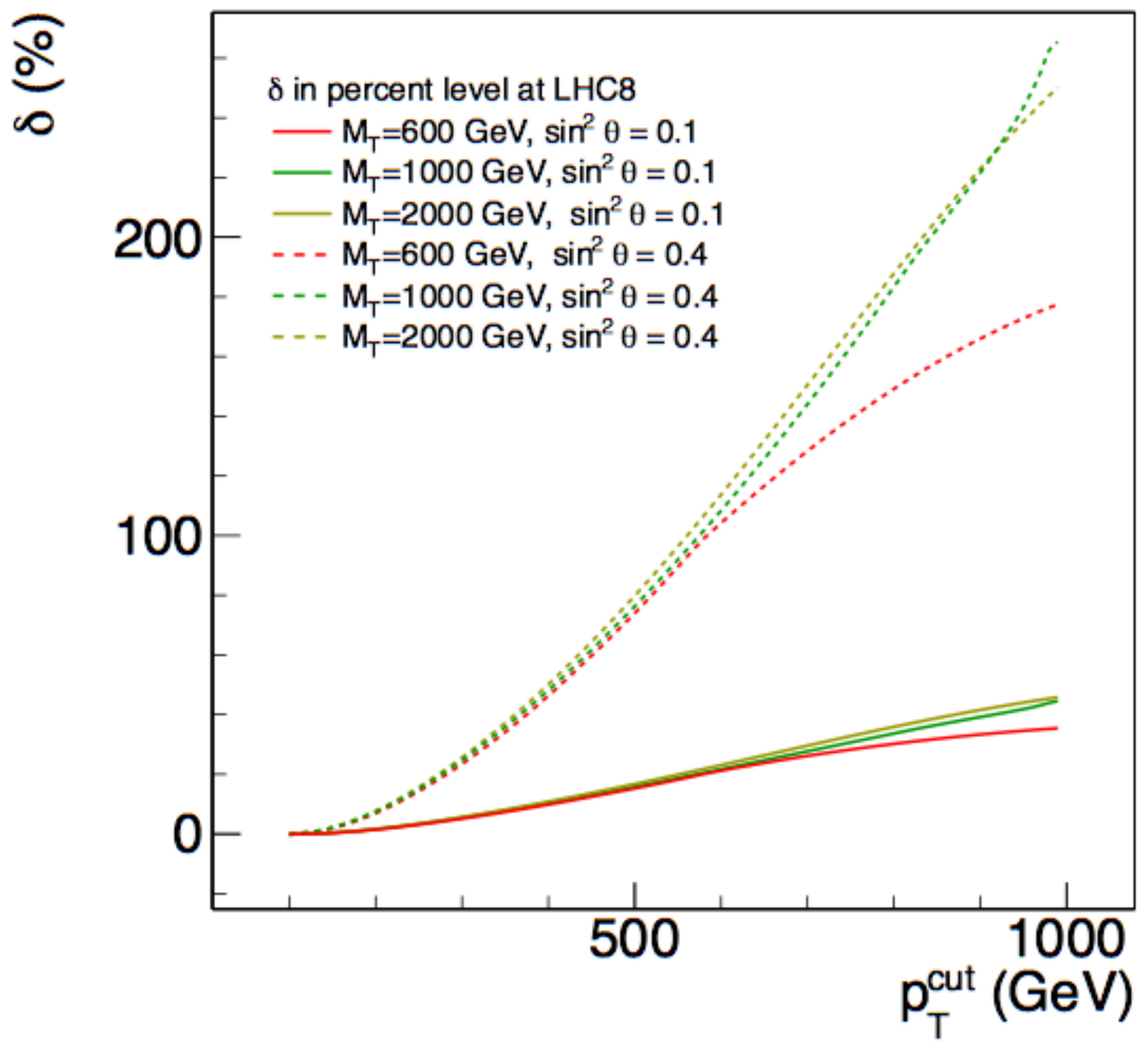
Is the Higgs boson coupling to gluons — a loop process — truly Standard Model like? A possible interplay between a point-like component and modifications of the top-Yukawa. Interesting to disentangle the two possibilities. Can be done if Higgs production at high transverse momentum is measured.

$$\frac{m_t}{v} t\bar{t}H \to -\kappa_g \frac{\alpha_s}{12\pi v} G^a_{\mu\nu} G^{a,\mu\nu} H + \kappa_t \frac{m_t}{v} t\bar{t}H$$



$$\sigma_H \sim \frac{\alpha_s^2}{v^2} (\kappa_g + \kappa_t)^2$$

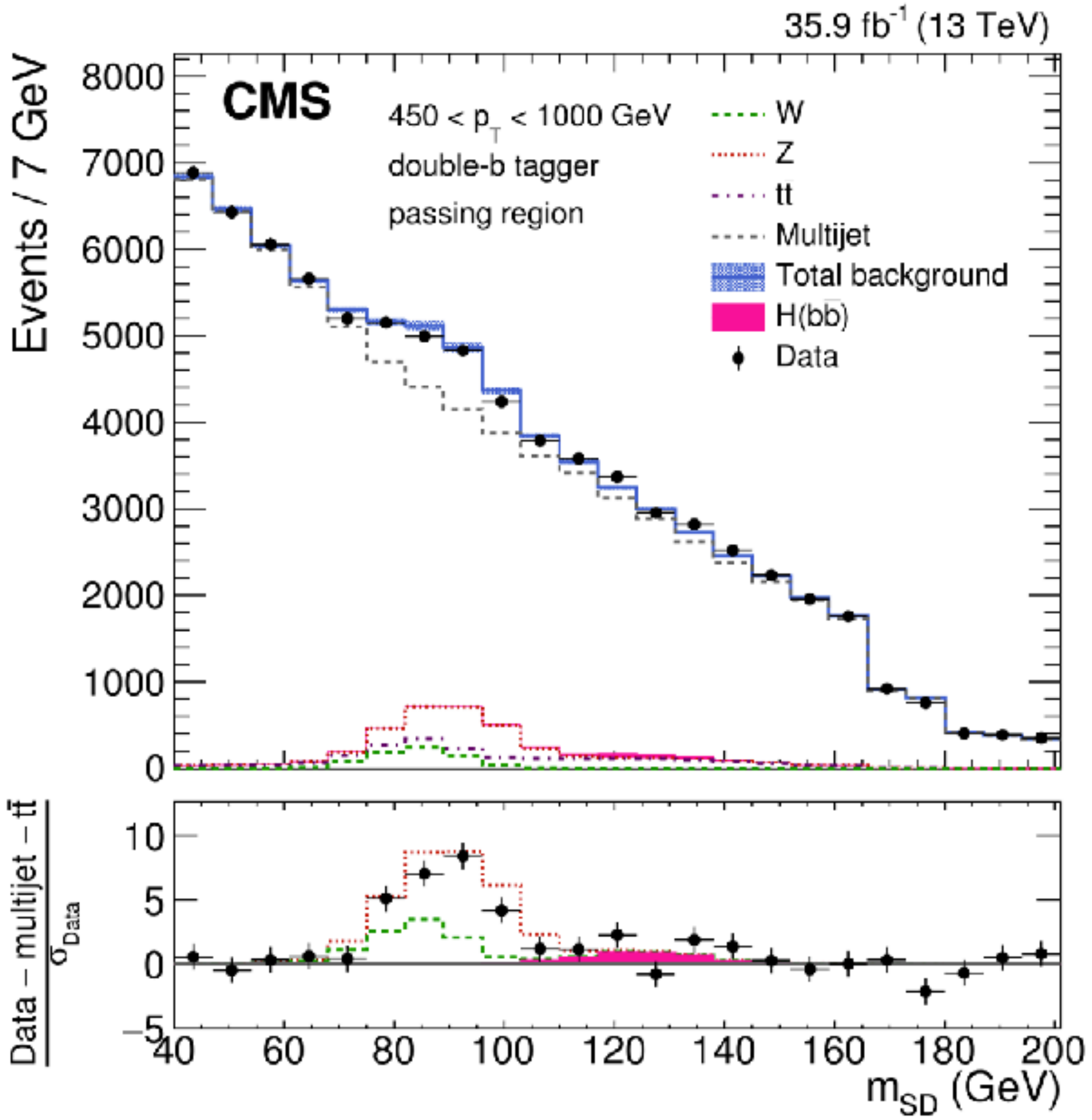
$$\frac{\sigma_H}{\mathrm{d}p_\perp^2} \sim \frac{\sigma_0}{p_\perp^2} \times \begin{cases} (\kappa_g + \kappa_t)^2, & p_\perp^2 < 4m_t^2, \\ \left(\kappa_g + \kappa_t \frac{4m_t^2}{p_\perp^2}\right)^2, & p_\perp^2 > 4m_t^2. \end{cases}$$



Contribution of fourth-generation quarks to H+j production in dependence of their masses, coupling and Higgs transverse momentum.

HIGGS PRODUCTION AT HIGH PT

CMS has already performed the measurement of the high-p_t Higgs production. Decays of the Higgs to b-pairs, are identified using the substructure techniques. Boosted Z is observed with 5 sigma significance. Higgs signal with a (much) smaller significance -- but this is a very inspiring beginning !



PRL 120, 071802 (2018)

$$\sigma_H \times \mathrm{Br}(H \to b\bar{b}) = 74 \pm 50 \text{ fb}$$

Systematic source	W/Z	H
Integrated luminosity	2.5%	2.5%
Trigger efficiency	4%	4%
Pileup	<1%	<1%
$N_2^{1,\mathrm{DDT}}$ selection efficiency	4.3%	4.3%
Double-b tag	4% (Z)	4%
Jet energy scale/ resolution	10/15%	10/15%
Jet mass scale (p_T)	0.4%/100 GeV (p_T)	0.4%/100 GeV (p_T)
Simulation sample size	2–25%	4–20% (GGF)
H p_T correction	…	30% (GGF)
NLO QCD corrections	10%	…
NLO EW corrections	15–35%	?
NLO EW W/Z decorrelation	5–15%	

$$\frac{m_t}{v} t\bar{t}H \to -\kappa_g \frac{\alpha_s}{12\pi v} G^a_{\mu\nu} G^{a,\mu\nu} H + \kappa_t \frac{m_t}{v} t\bar{t}H$$

Need to understand theoretical aspects of this process better ! Indeed, a 60 percent enhancement of the cross section at high transverse momentum — is it heavy top partners or just a QCD effect?

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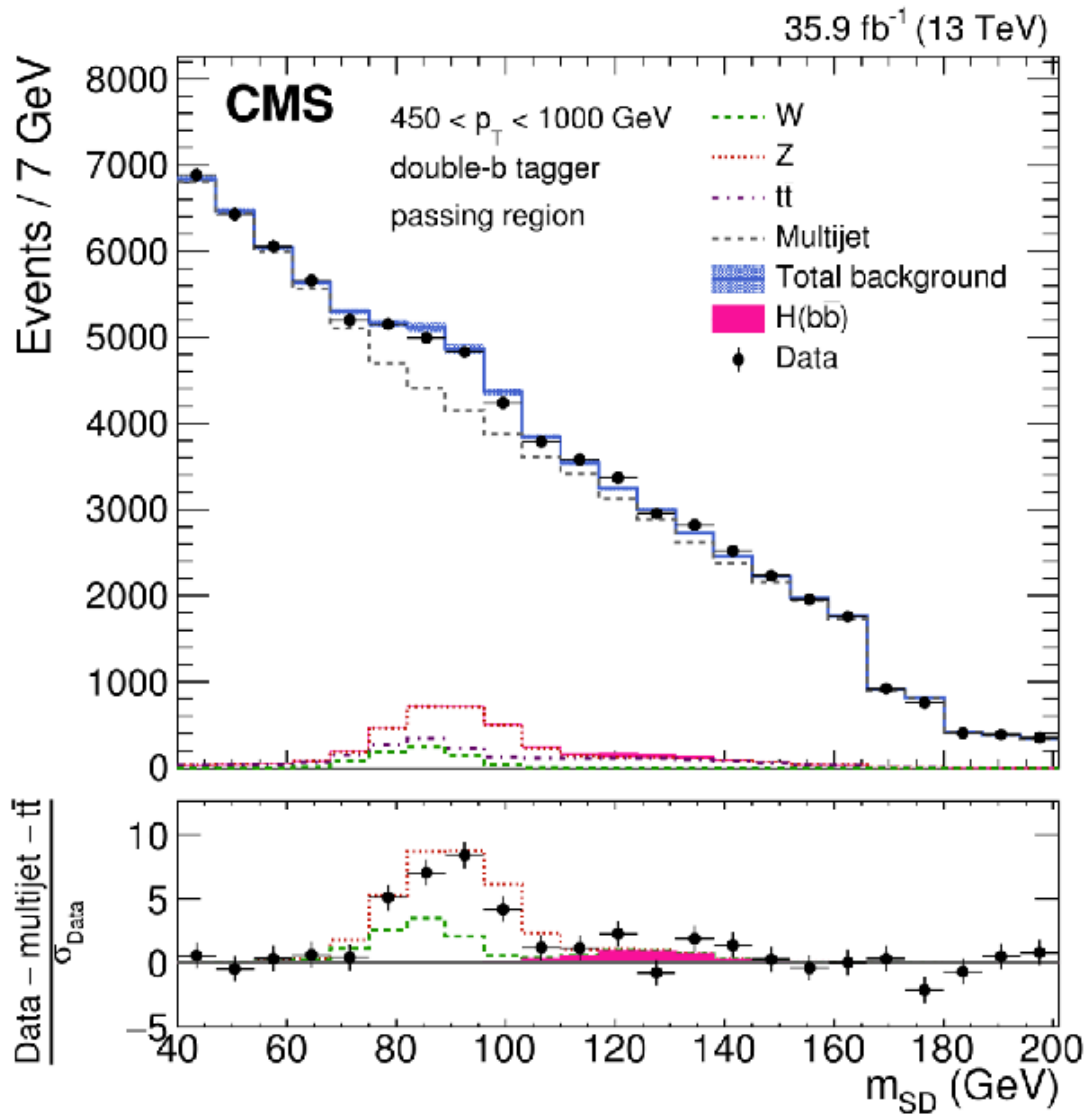

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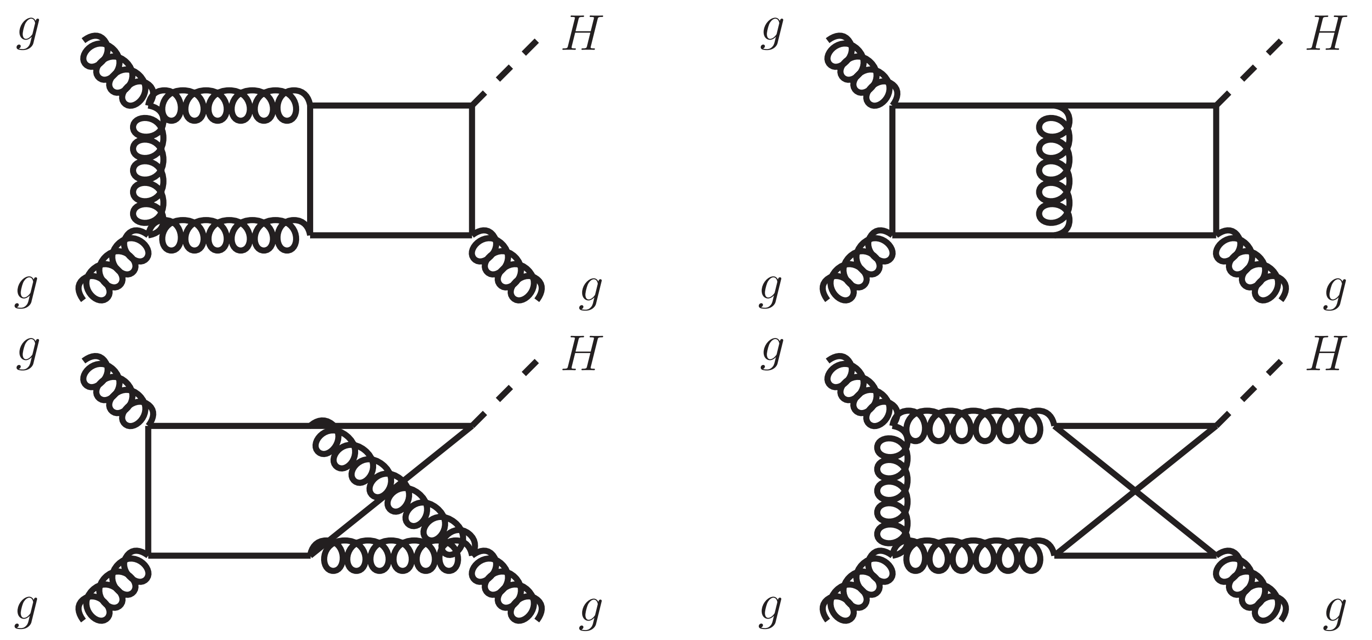
Simulation sample size	2–25%	4–20% (GGF)
NLO EW corrections		
NLO EW W/Z decorrelation		

TO MAKE PROGRESS, WE NEED TO UNDERSTAND HOW TO COMPUTE NON-TRIVIAL (LOOPS+MASSES) FEYNMAN INTEGRALS



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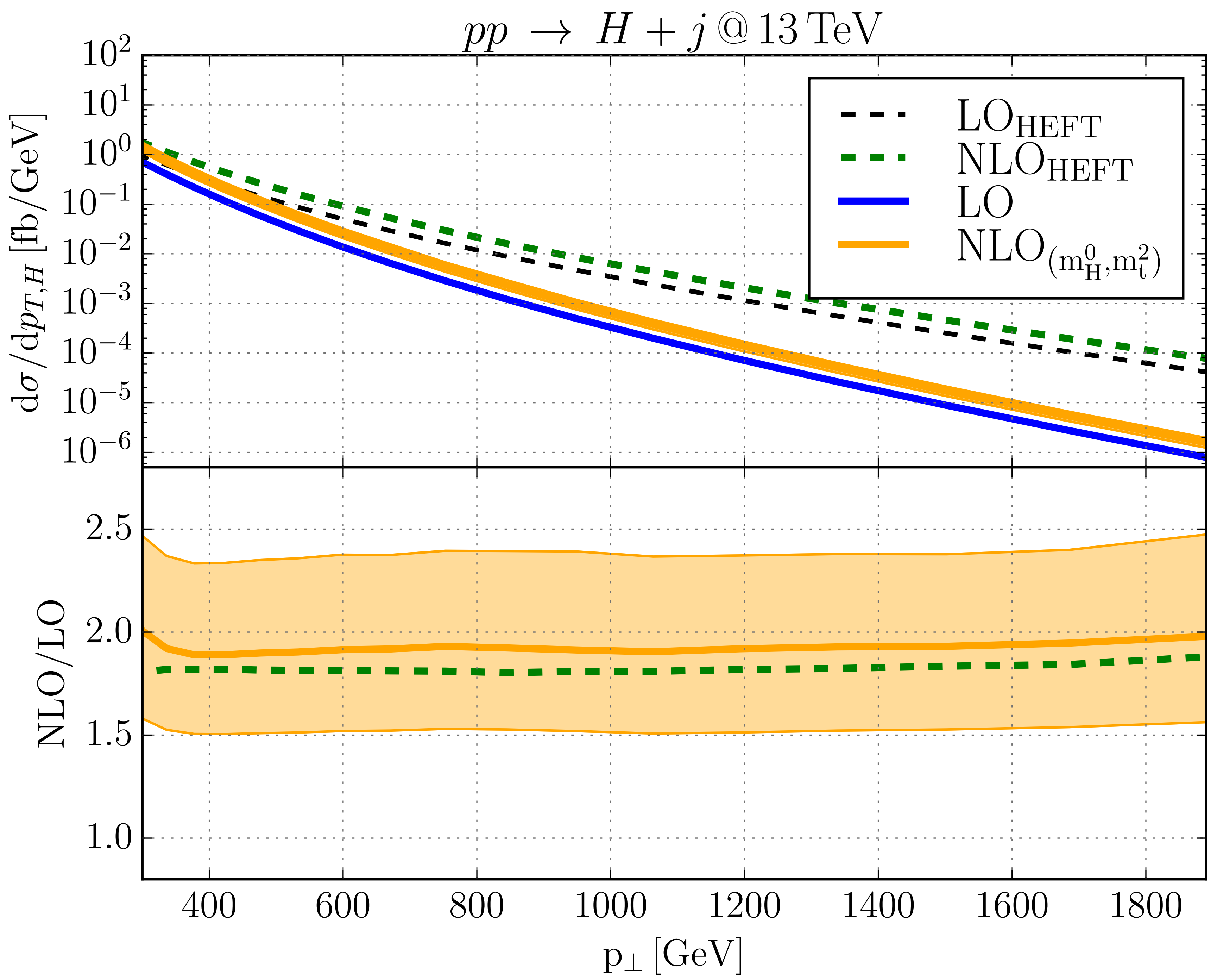
HIGGS PRODUCTION AT HIGH PT



-K-factors are large but largely p_t-independent;

- K-factors in the full theory are about 6 percent larger than in HEFT;

- Large differences compared to the central values of the experimental result.

$$\sigma_H \times \mathrm{Br}(H \to b\bar{b})|_{\rm th,NLO} = 7.5 \text{ fb} + 20\% \text{ NNLO ?} \quad p_\perp > 450 \text{ GeV}$$

$$\frac{m_t}{v} t\bar{t}H \to -\kappa_g \frac{\alpha_s}{12\pi v} G^a_{\mu\nu} G^{a,\mu\nu} H + \kappa_t \frac{m_t}{v} t\bar{t}H$$

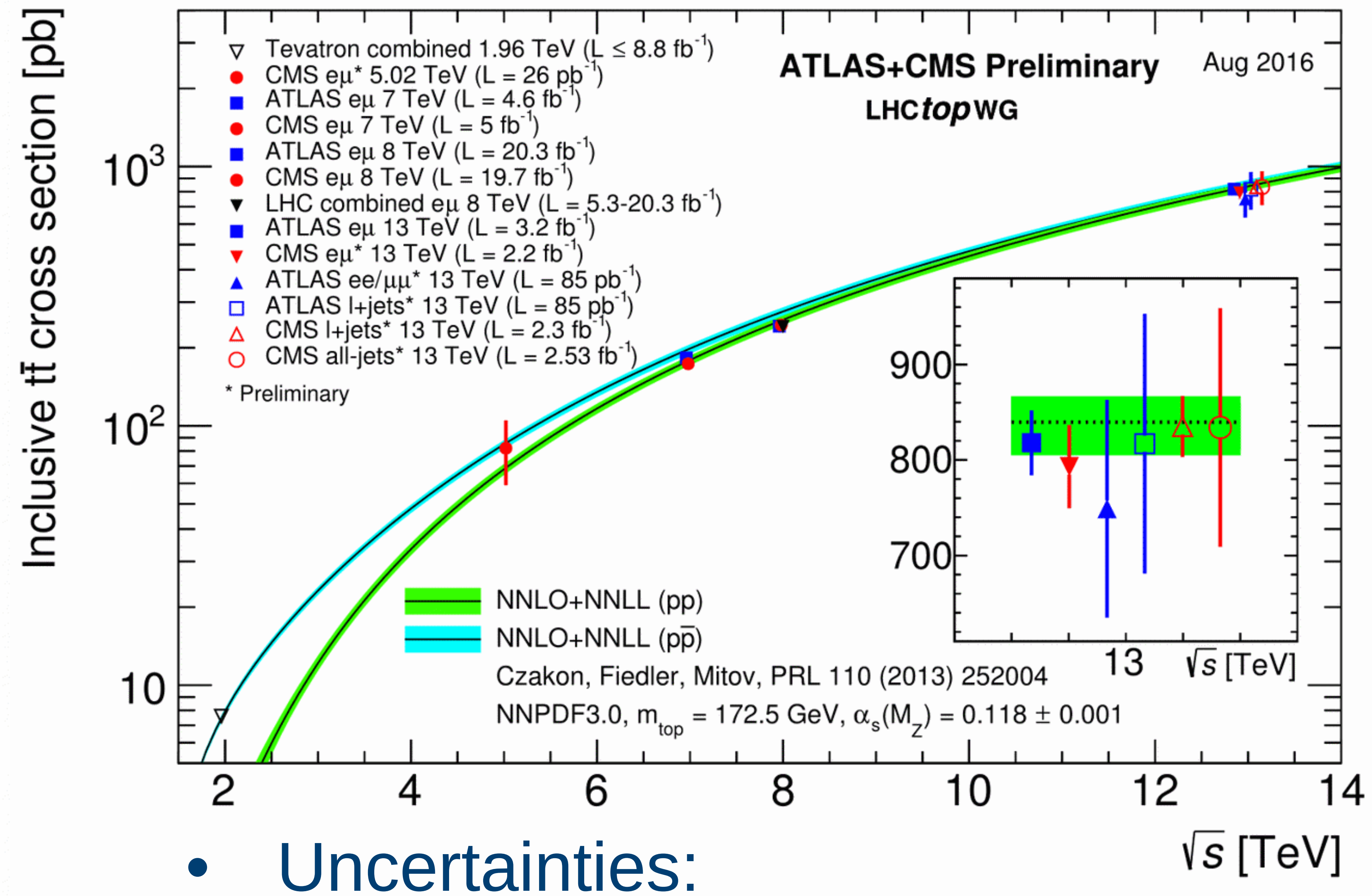
	LO$_{\rm HEFT}$ [fb]	NLO$_{\rm HEFT}$ [fb]	K	LO [fb]	NLO [fb]	K
$p_\perp >$ 400 GeV	$33.8^{+44\%}_{-29\%}$	$61.4^{+20\%}_{-19\%}$	1.82	$12.4^{+44\%}_{-29\%}$	$23.6^{+24\%}_{-21\%}$	1.90
$p_\perp >$ 450 GeV	$22.0^{+45\%}_{-29\%}$	$39.9^{+20\%}_{-19\%}$	1.81	$6.75^{+45\%}_{-29\%}$	$12.9^{+24\%}_{-21\%}$	1.91
$p_\perp >$ 500 GeV	$14.7^{+44\%}_{-28\%}$	$26.7^{+20\%}_{-19\%}$	1.81	$3.80^{+45\%}_{-29\%}$	$7.28^{+24\%}_{-21\%}$	1.91
$p_\perp >$ 1000 GeV	$0.628^{+46\%}_{-30\%}$	$1.14^{+21\%}_{-19\%}$	1.81	$0.0417^{+47\%}_{-30\%}$	$0.0797^{+24\%}_{-21\%}$	1.91

Table: Inclusive cross sections and K-factors for $pp \to H$+jet at $\sqrt{S}$=13 TeV in the SM and in the infinite top

K. Kudashkin, J. Lindert, K.M., C. Wever; S. Jones, M. Kerner, J. Luisoni; T. Neumann

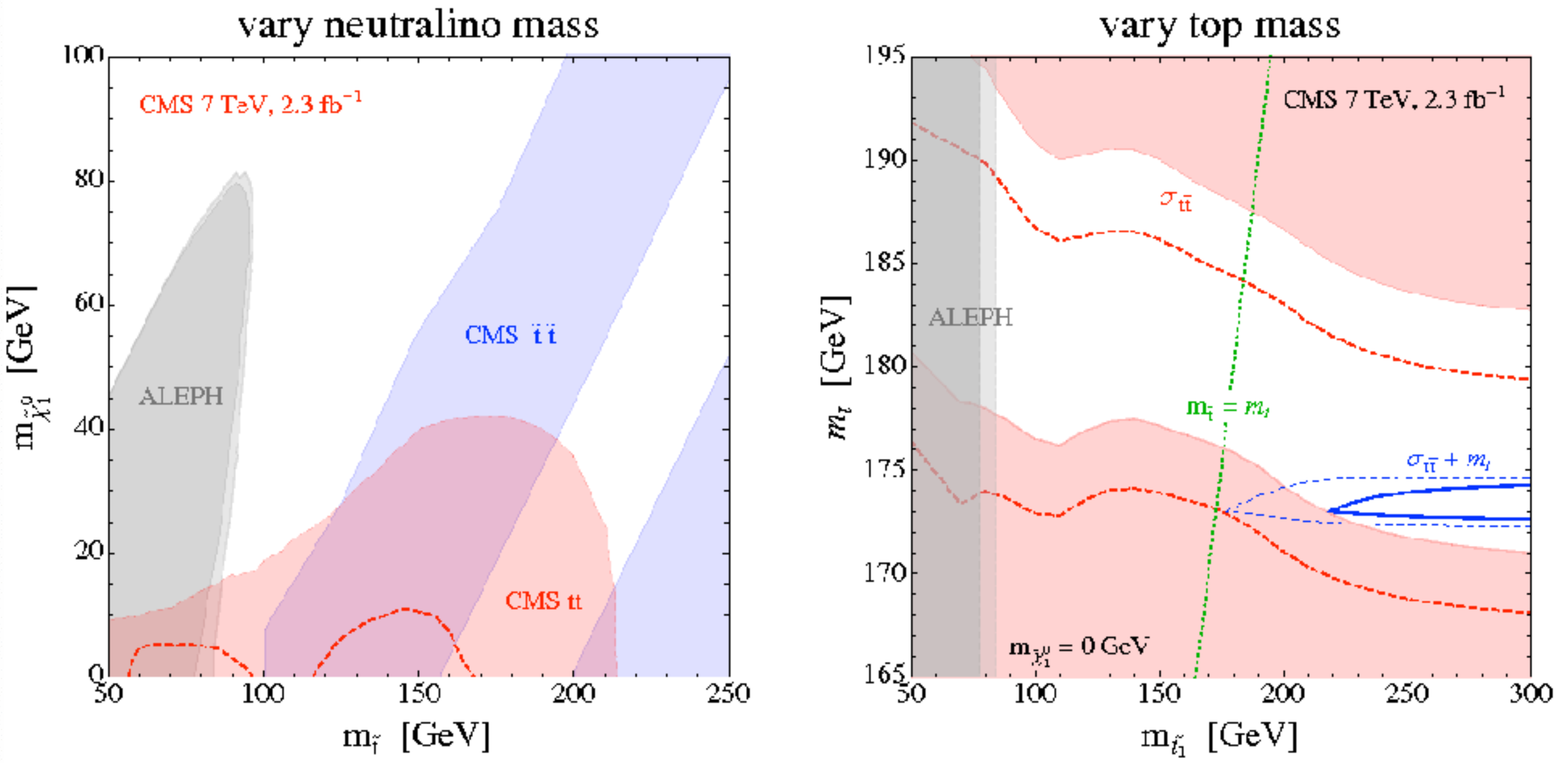
PRECISION PHYSICS WITH TOP QUARKS

Improved calculations of top quark pair production cross section can be used to extract the top quark mass from the inclusive cross section, constrain the strong coupling constant and parton distribution functions. In addition, they are instrumental for excluding the existence of ``stealthy stops'', i.e. supersymmetric partners of the top quark whose masses are very close to m_t.



- Uncertainties:

Scales	~ 3-4%
pdf	~ 2-3%
α_S	~ 1.5%
m_{top}	~ 3%



$$\sigma_{\tilde{t}\tilde{t}} \sim 0.14\,\sigma_{tt} \qquad m_{\tilde{t}} = m_t \qquad \tilde{t} \to \tilde{Z}t$$

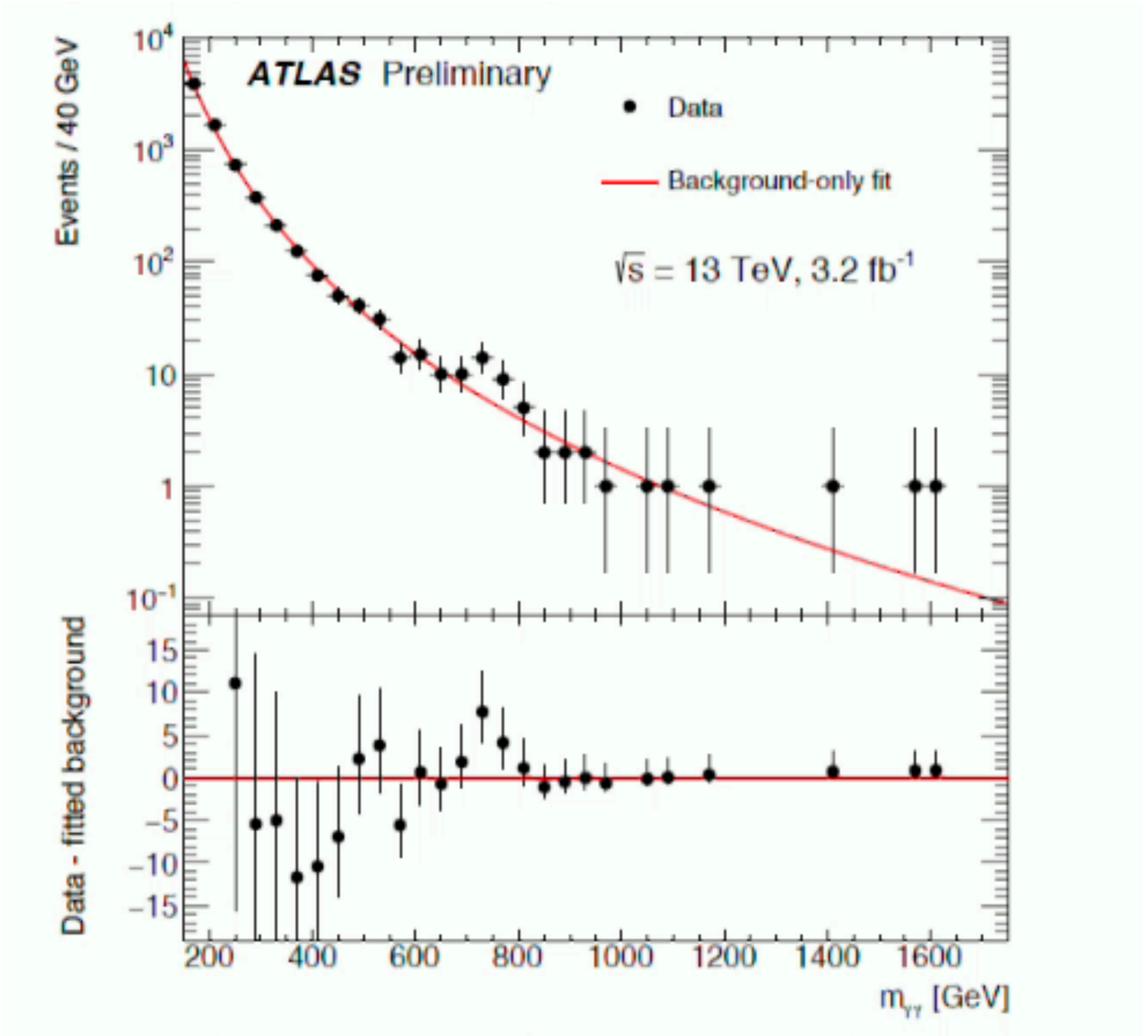
$$\sigma_{\tilde{t}\tilde{t}} < \sqrt{\delta\sigma^2_{t\bar{t},\mathrm{exp}} + \delta\sigma^2_{t\bar{t},\mathrm{th}}}$$

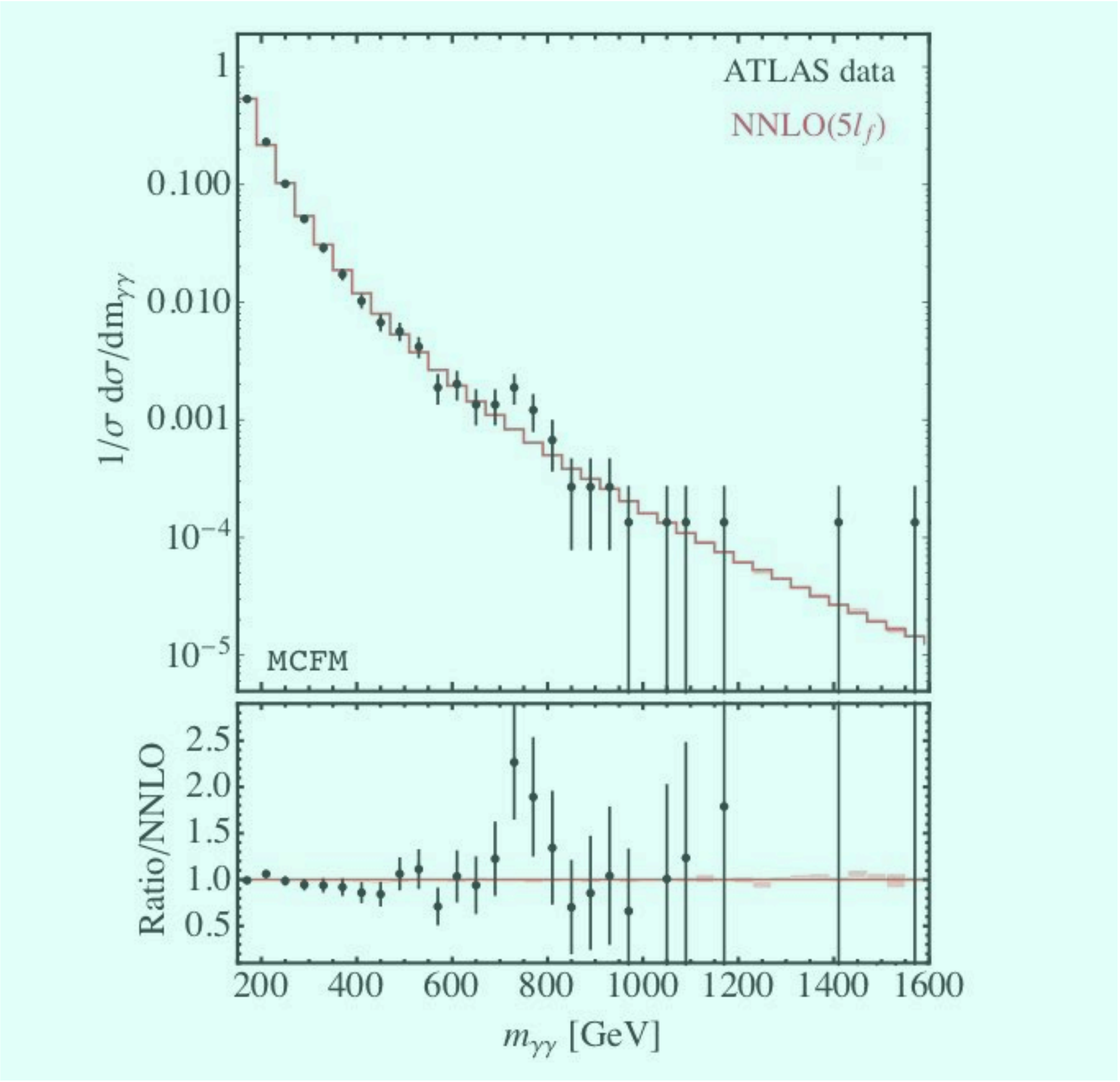
Czakon, Heymes, Mitov

Czakon, Mitov, Ruderman, Papucci, Weiler

A 750-GEV RESONANCE AND THE SHAPE OF THE DI-PHOTON SPECTRUM

NNLO QCD (normalized) predictions for the di-photon invariant mass distribution compare perfectly well with ATLAS data (3.2/fb, 13 TeV), in spite of the ambiguities in defining a ``photon''. No need to fit the background to ``discover'' a 750 GeV resonance!

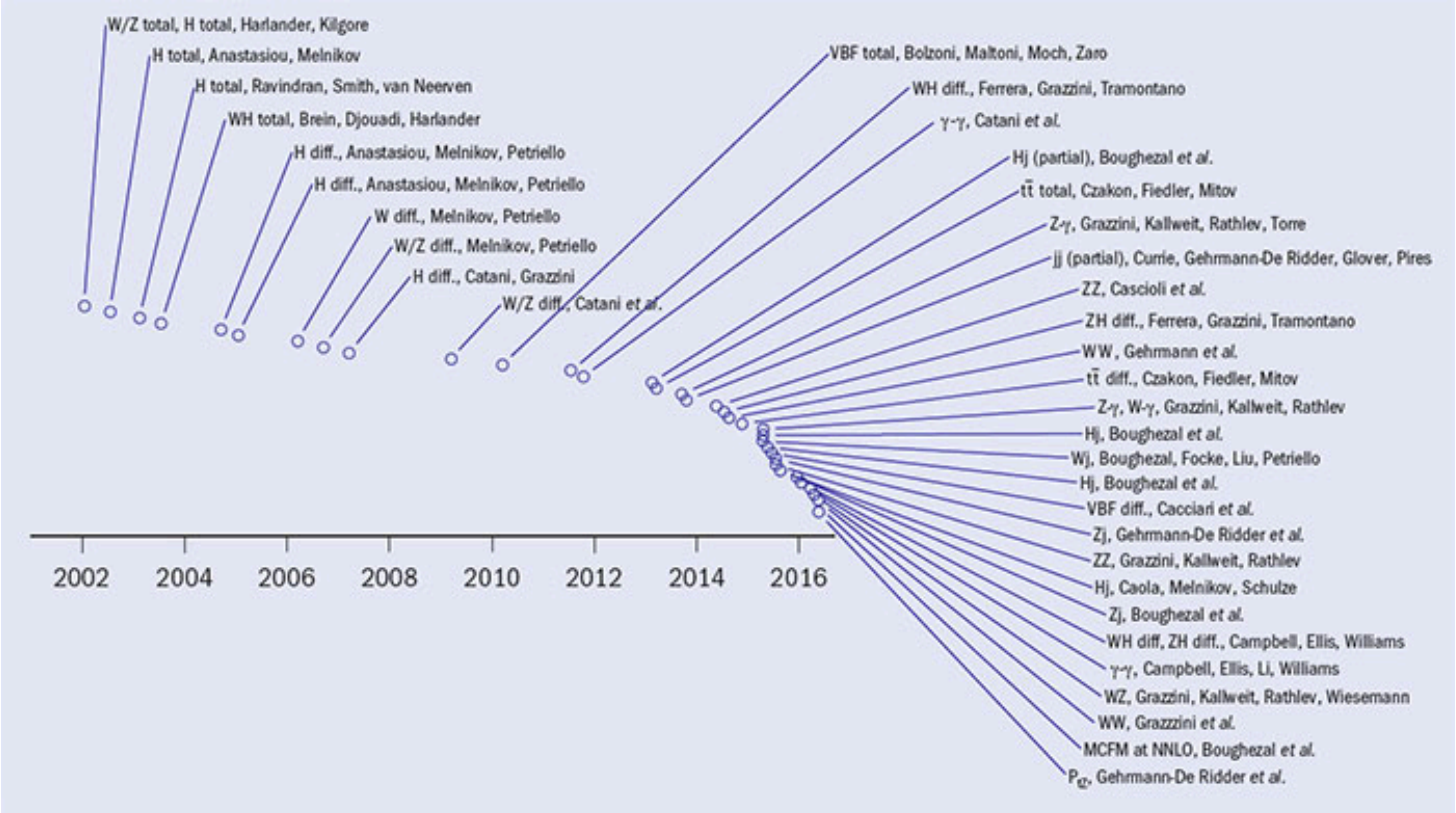




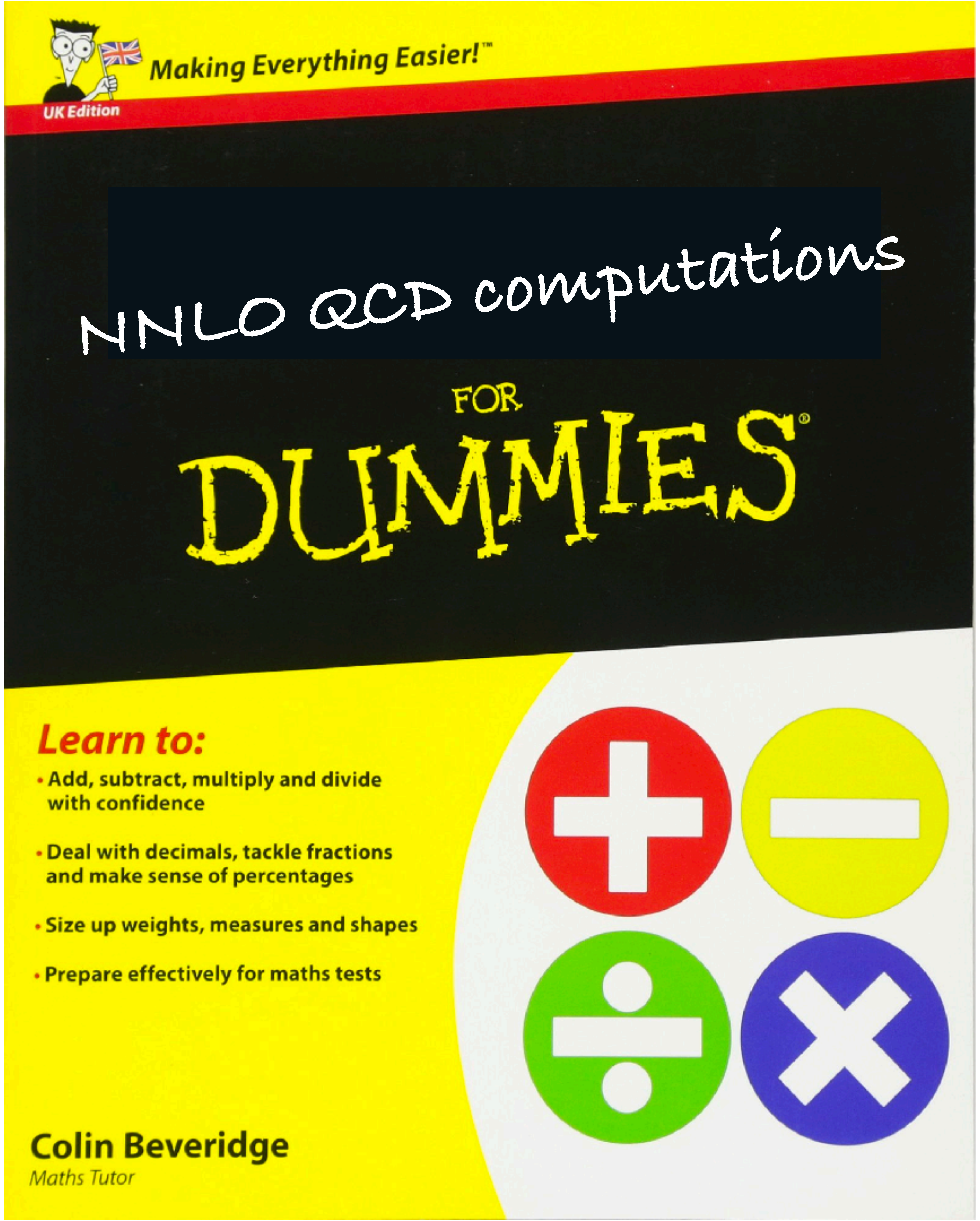
Campbell, Ellis, Williams

NNLO QCD PREDICTIONS FOR MAJOR LHC PROCESSES

Remarkable pace of progress in recent years — most of the relevant 2 ->2 LHC processes (jj, H+j, V+j, tT, VH) are known through NNLO QCD. 2->3 processes (3j, 2gamma+j, ZbB, tTH). is the next frontier.



G. Salam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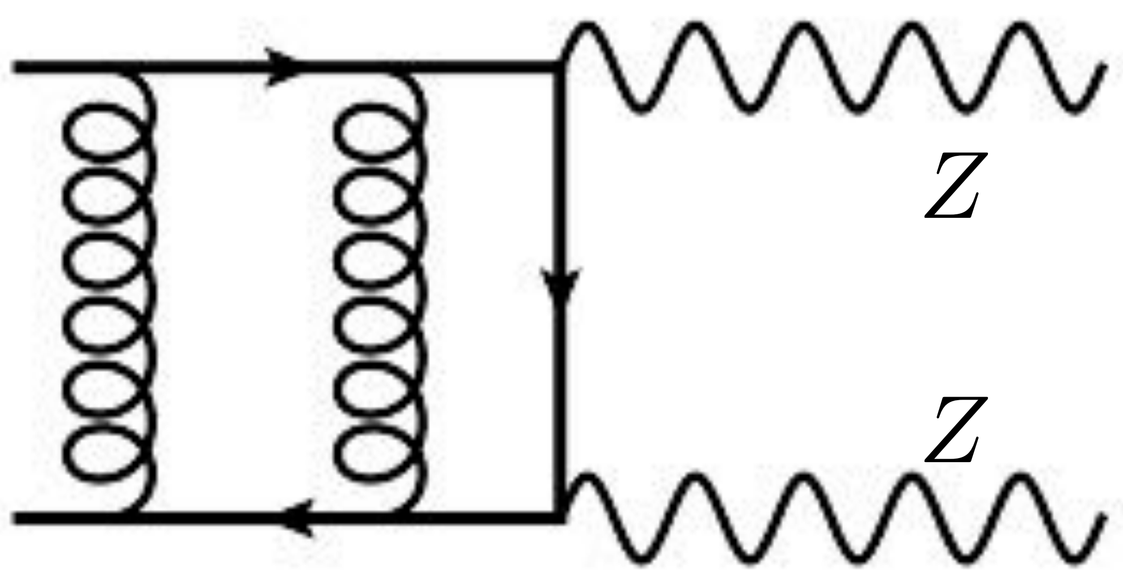
PERTURBATIVE QCD: REAL AND VIRTUAL CORRECTIONS

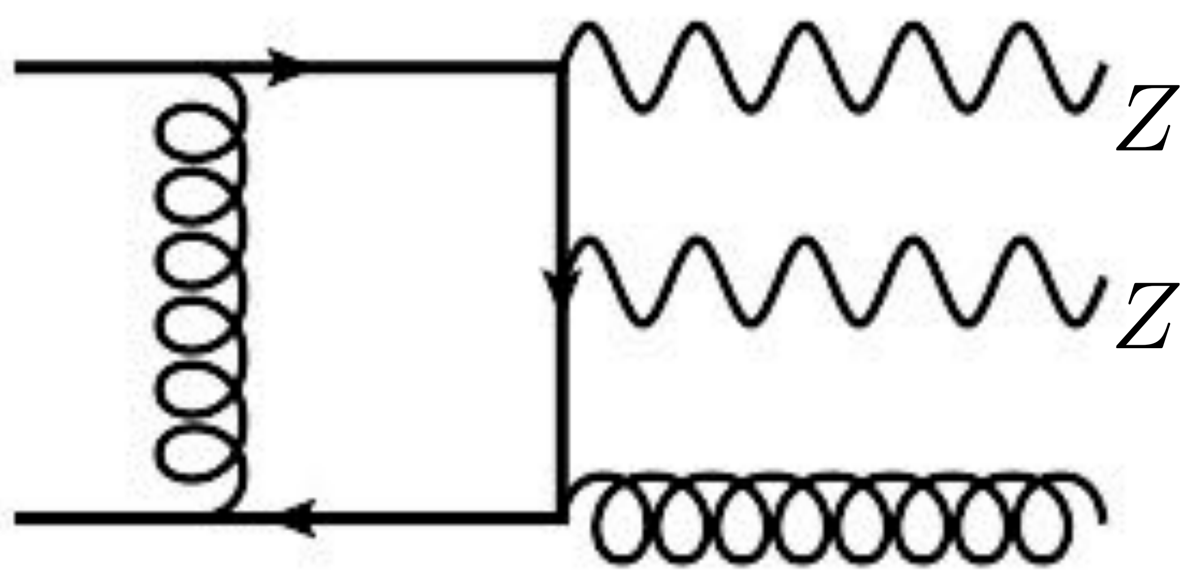
Processes with fixed number of final state particles are sensitive to long-distance physics, computation of QCD corrections requires us to consider both virtual and real-emission corrections.

Virtual corrections is difficult to compute.

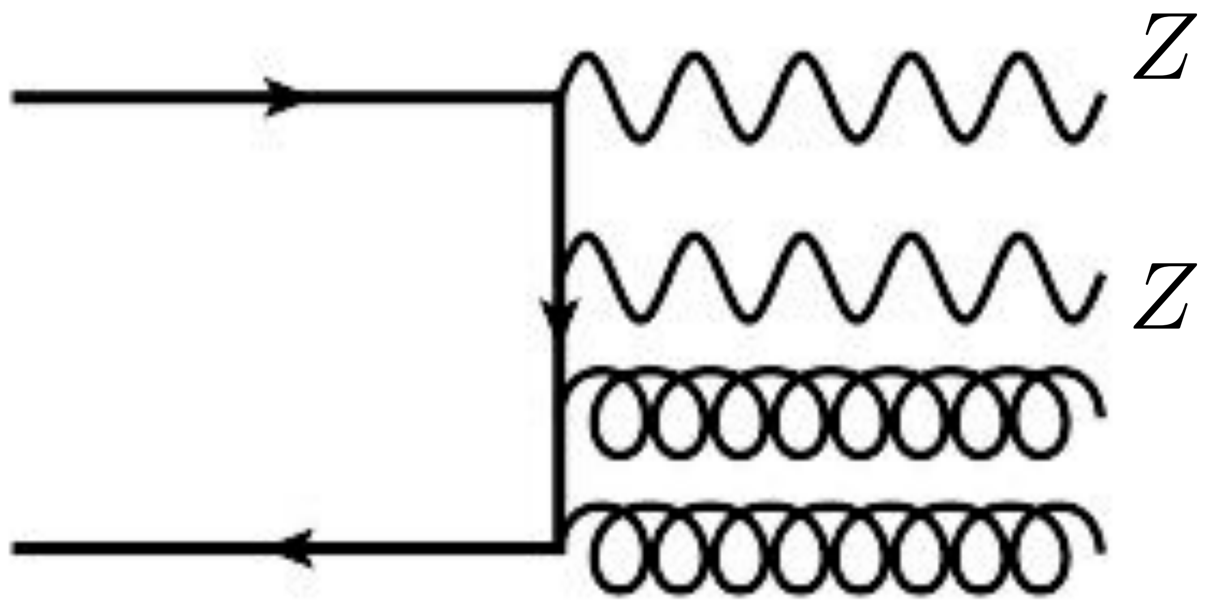
It is difficult to combine real and virtual contributions together, to obtain physical cross sections.



Virtual corrections



Real and virtual



Real corrections

A NNLO COMPUTATION FOR ZZ PRODUCTION REQUIRES US TO COMBINE QQ -> ZZ, QQ -> ZZ+G AND QQ -> ZZ+GG PARTONIC PROCESSES

VIRTUAL CORRECTIONS

For processes of interest we have to deal with very large number of very complicated Feynman integrals (hundreds diagrams, thousands of integrals etc.); a one-by-one integration is unthinkable. The question, therefore, is how to simplify the amount of analytic work that is required.

Key ideas:

1) integration-by-parts: reduces the number of independent integrals to be computed;

2) generalized unitarity: allows us to work with on-shell amplitudes that are simpler than diagrams;

3) differential equations for Feynman integrals: better to solve a differential equation than to compute an integral;

4) numerical computation of Feynman integrals: the ugly future..

VIRTUAL CORRECTIONS: REDUCING THE NUMBER OF INTEGRALS

Computation of integrals through derivatives is referred to as ``integration-by-parts''. The starting point is simple, even embarrassingly simple, but the consequences are profound.

Chetyrkin, Tkachov

$$0 = \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{\partial}{\partial k_\mu} \left[\frac{l^\mu}{D_1^{a_1} D_2^{a_2} D_3^{a_3} \ldots} \right] \Rightarrow \sum_k c_k(s_{ij}, m_i) I_k = 0$$

Conceptually, a reduction to master integrals became a straightforward exercise in linear algebra after ``Laporta algorithm'' was invented.

Laporta

Computer-algebra packages can do a reduction for you. FIRE, REDUZE, LITERED, KIRA

Smirnov, von Manteufffel, Lee, Maierhoefer, Usovitsch, Uwer

$$I_k = \sum c_{kj}(s_{ij}, m_n^2)\, \tilde{I}_j$$

For multi-scale problems (many particles with different masses, many kinematic scales), the reduction to master integrals becomes exceedingly difficult problem because algebraic complexity increases dramatically.

However, these complex cases are needed! Indeed, 2 -> 3 two-loop partonic amplitudes is a new frontier, relevant for 3-jet production, vector boson + 2 jets, Higgs + 2 jets, diphoton + jets, ttH, etc.

Many new ideas (e.g. Baikov representation, "finite fields reconstruction") on how to force the reductions to work in practice, also in these complex cases; very active field of research.

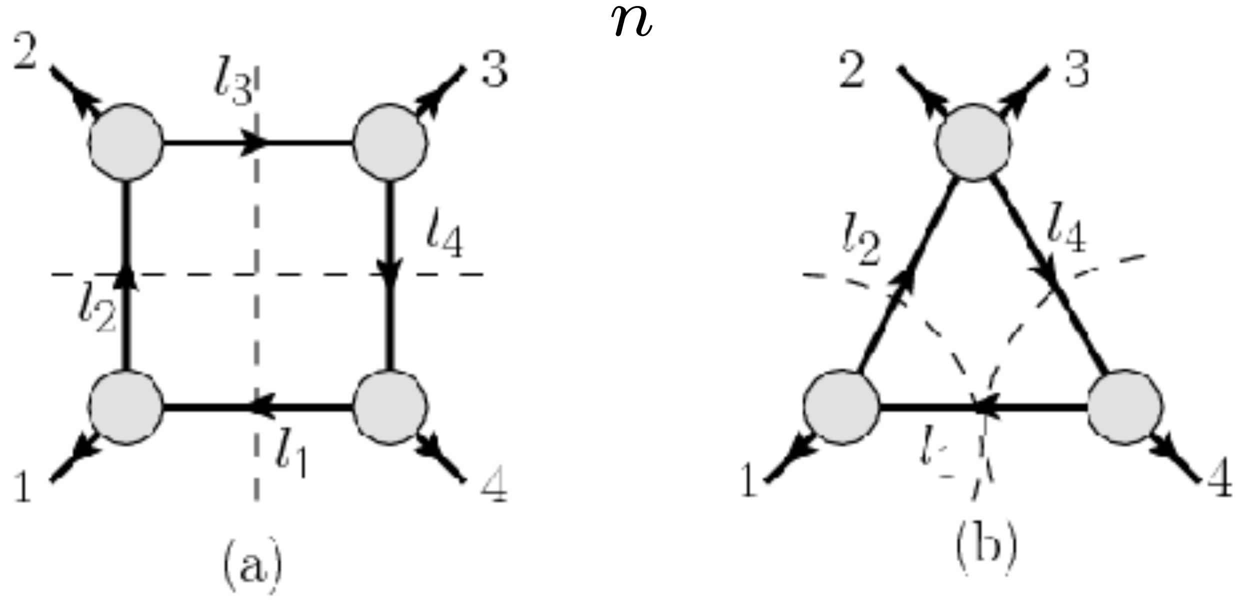
VIRTUAL CORRECTIONS: REDUCTION THROUGH UNITARITY

Generalized unitarity provides a different approach to the reduction to master integrals; reduction coefficients are reconstructed from on-shell scattering amplitudes . Very successful method at one-loop; attempts to generalise to two- loops.

Recent progress with the evaluation of planar (large N_c) contribution to five-gluon two-loop amplitude. An impressive proof of concept that unitarity works at two-loops but still far from a real computation of the full scattering amplitude and e.g. the phenomenology of the three-jet NNLO cross sections.

$$T^+_{fi} - T_{if} = \sum_n T^+_{fn} T_{ni} \qquad f(s) = \int_{s_{\rm th}}^{\infty} \frac{{\rm d}s \, {\rm Im} f(s)}{s - s_0 - i\delta}$$

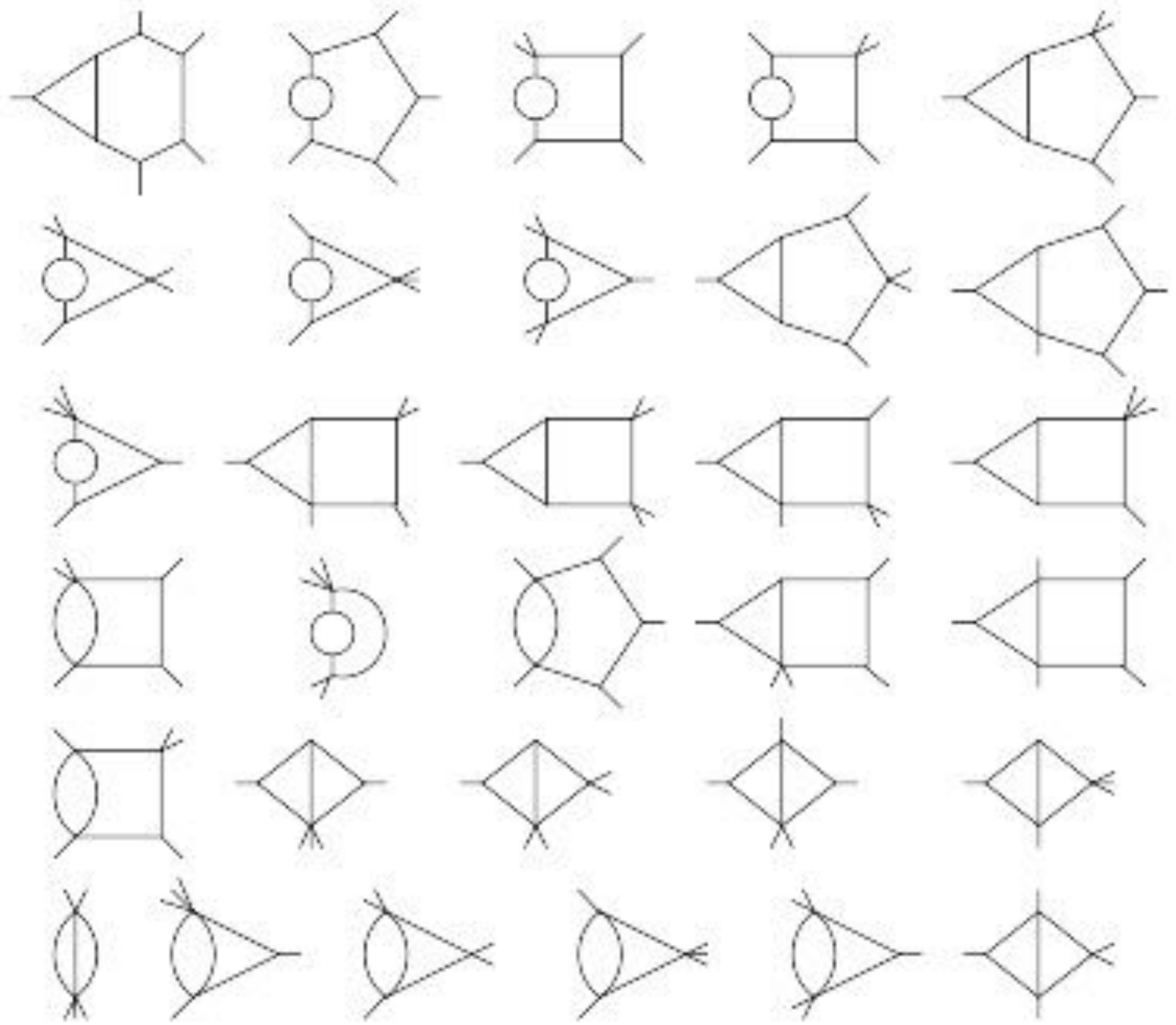
$$T_{fi} = \sum_n^{\infty} c^n_{fi} \, I_n(s,t,u,...,m_1,..)$$



$$\mathcal{A}(1^+ ... i^- ... j^-, ... n^+) = i(-g)^{n-2} \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle .... \langle n-1 n \rangle}$$

Bern, Dixon, Kosower ; Britto, Cachazo, Feng, Ossola, Papadopoulos, Pittau; Elllis, Giele, Kunstz

$0 \to 5g$ @ NNLO



Abreu, Cordero, Ita, Page, Zeng
Badger, Bronnum-Hansen, Hartano, Peraro

VIRTUAL CORRECTIONS: INTEGRALS

Master integrals need to be computed after the reduction; this can done either analytically or numerically.

Analytic computations were a preferable choice for a long time (transparent results, fast and numerically-exact evaluations). The complexity of analytic results increases dramatically with the number of scales (kinematic invariants and particle masses).

Analytic computations in recent years were mostly performed using differential equations (that follow from IBP's) in the canonical form.

We benefited from a better understanding of particular special functions — the so-called Goncharov polylogarithms — that appear in many solutions of differential equations for various Feynman integrals.

$$s_i \frac{\partial}{\partial s_i} \vec{I} = \epsilon \hat{A}(\{s\}) \vec{I},$$

Henn

$$G(\{a_n, \vec{a}_{n-1}\}, x) = \int_0^x \frac{\mathrm{d}t}{t - a_n} G(\{\vec{a}_{n-1}\}, t), \qquad G(\{\vec{0}_n\}, x) = \frac{1}{n!} \log^n x.$$

Goncharov

There are many relevant cases where canonical form can not be achieved (ttH, ttZ, Zbb etc.). Different class of functions is needed (elliptic integrals appear). What are these functions and what are their properties is the topic of a very active exploration currently.

$$K(x, a) = \int_0^x \frac{\mathrm{d}t}{\sqrt{(1 - t^2)(1 - a t^2)}}$$

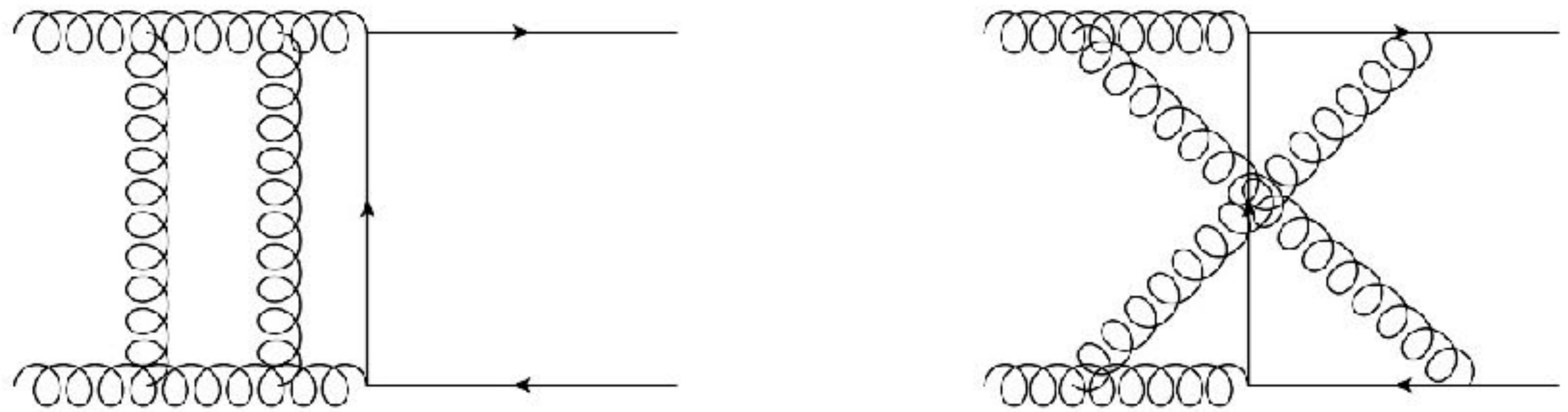
Remiddi, Tancredi, Adams, Bogner, Weinzierl,, Duhr, Broedel, Dulat, Penante

VIRTUAL CORRECTIONS: INTEGRALS

Interesting physics requires higher-order computations for generic mass assignments (ttH, ttV, Zbb etc.); hard to imagine that analytic computations will continue to play a leading role — sooner or later we will be forced to go numerical!

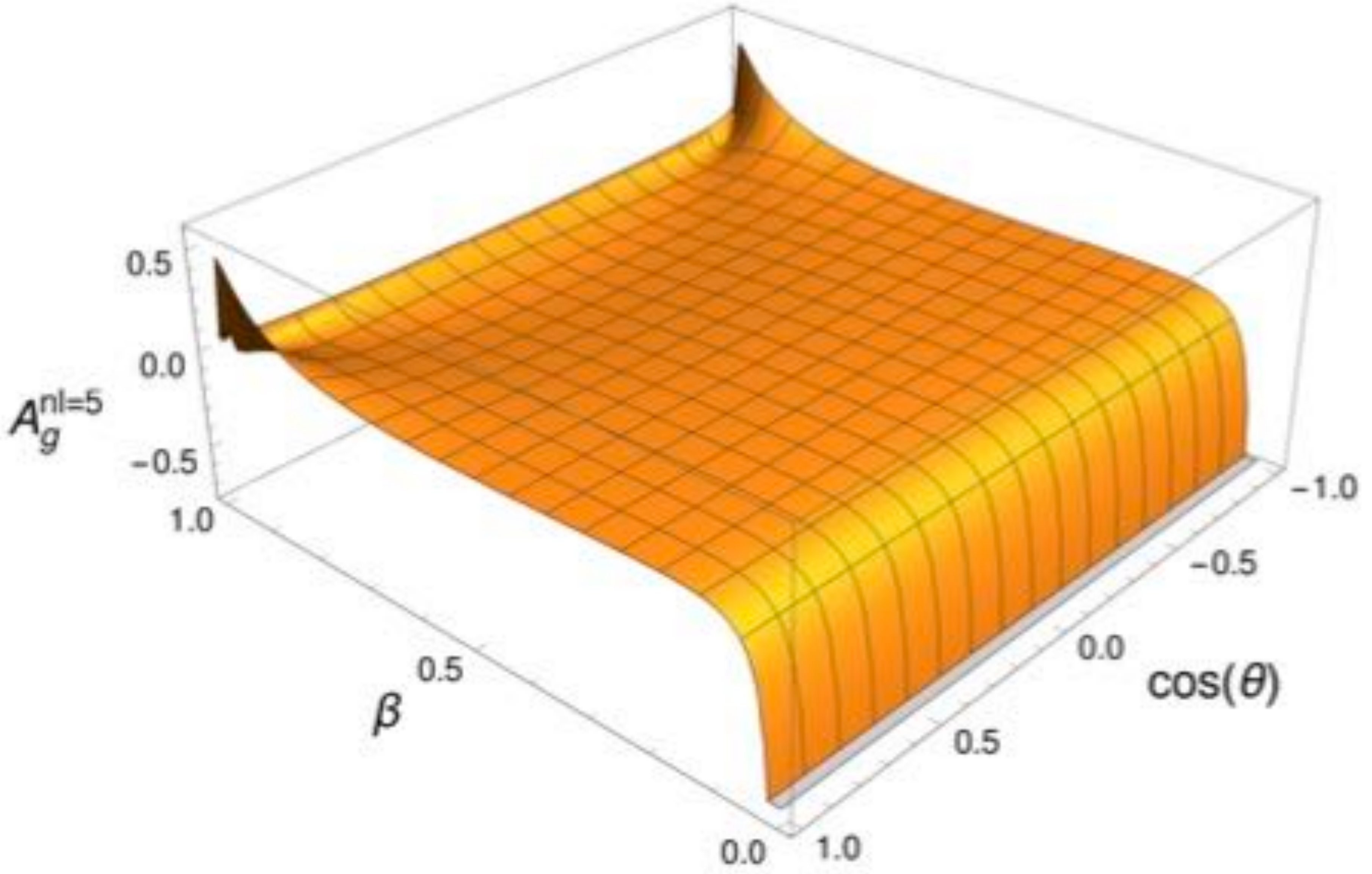
There are two examples of very successful application of the numerical methods for the computation of loop integrals.

Top quark pair production $gg \to t\bar{t}$



$$m_s \frac{\partial}{\partial m_s} \vec{I} = \hat{A}_m(m_s, x, \epsilon)\vec{I}$$

$$x \frac{\partial}{\partial x} \vec{I} = \hat{A}_x(m_s, x, \epsilon)\vec{I} \qquad m_s = \frac{m_t^2}{s}, \quad x = \frac{t}{s}$$



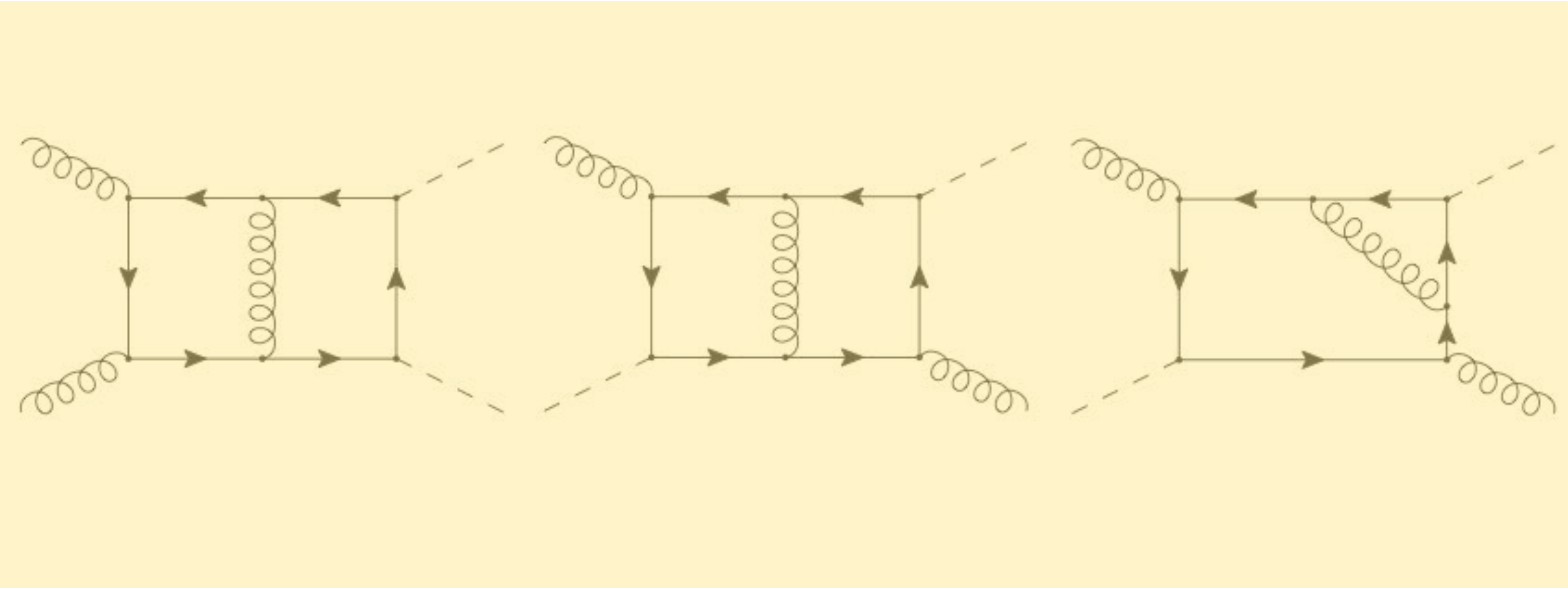
Chen, Czakon, Poncelet

Boundary conditions are computed at very high energy, for nearly massless quarks.

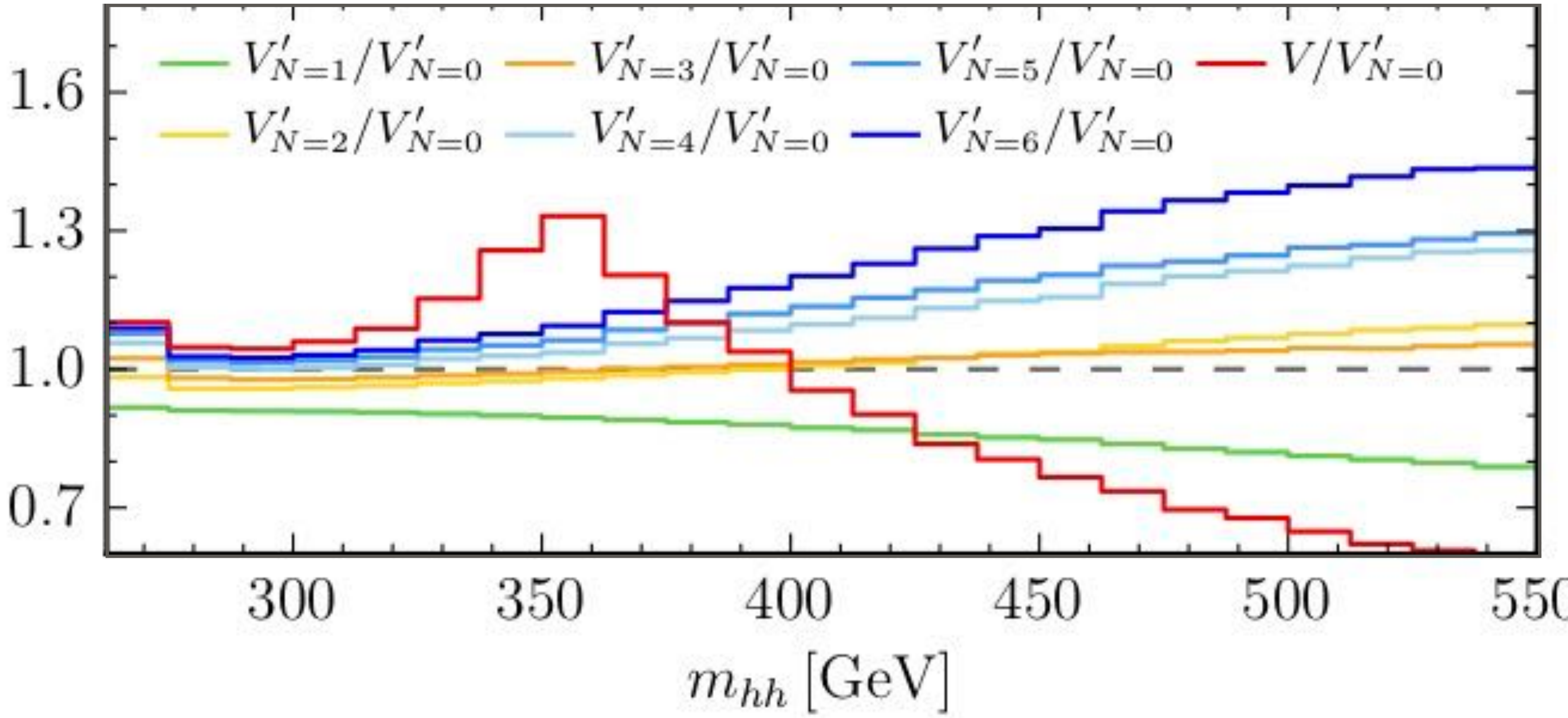
VIRTUAL CORRECTIONS: INTEGRALS

Master integrals can also be computed upon numerical integration over Feynman parameters (SecDec). This method has been successfully applied to double Higgs and Higgs + jet production at the LHC with the full top mass dependence.

$$\vec{I} = \int_0^1 .... \int_0^1 \mathrm{d}x_1 ... \mathrm{d}x_n \, \frac{\vec{N}(x_1, ..., x_n)}{D(x_1, x_2, ..., x_n)} \quad \longrightarrow \quad \vec{I} = \frac{\vec{a}_{-2}}{\epsilon^2} + \frac{\vec{a}_{-1}}{\epsilon} + \vec{a}_0$$



Double-Higgs production $\quad gg \to HH$



Borowka, Greiner, Heinrich, Kerner, Jones, Schenk, Zirke

INFRA-RED AND COLLINEAR SINGULARITIES

The key problem with combining real and virtual contributions is that a) they live in different phase-spaces and b) they diverge, when taken separately.

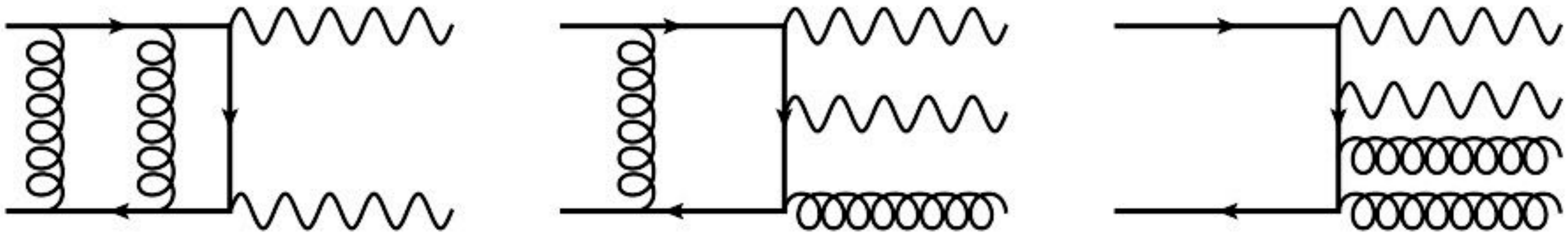
Virtual corrections are always integrated over the loop momenta; they explicitly exhibit infra-red and collinear singularities.

$$M = M_0 + \frac{\alpha_s}{2\pi}\left(I^{(1)} M_0 + M_{\rm fin}\right) \qquad I^{(1)} = \frac{e^{-\epsilon\gamma_E}}{2\Gamma(1-\epsilon)} \sum_i \frac{1}{T_i^2}\left(\frac{T_i^2}{\epsilon^2} + \frac{\gamma_i}{\epsilon}\right) \sum_{I \neq j} \vec{T}_i \cdot \vec{T}_j \left(\frac{\mu^2 e^{-i\lambda_{ij}\pi}}{2p_i \cdot p_j}\right)$$

Catani

$$\mathrm{d}\sigma_V \sim \int [\mathrm{d}p_f])(2\pi)^d \delta(p_f - p_i)\left(I_1 |\mathcal{M}_0|^2 + \mathcal{M}_0 \mathcal{M}_{\rm fin}\right) \mathcal{O}(p_f, p_i) \qquad d = 4 - 2\epsilon; \quad \epsilon \to 0.$$

Real corrections are finite in the bulk of a phase-space; to produce divergencies we need to integrate them over infra-red- and collinear-singular regions of phase-space where the final state looks similar to the final state of the tree-level process.



REAL EMISSION CONTRIBUTIONS

Real emission corrections are finite in the bulk of the allowed phase-space; infra-red and collinear divergencies only appear upon integration over energies and angles of the emitted partons.

Need to find a way that allows us to extract these divergencies without integration over resolved parts of the phase-space.

A number of methods have been developed to a point where ``mass production'' of NNLO results relevant for different physics is possible (in practice, limited to 2 ->1 and 2->2 since virtual amplitudes only exist for those).

Nevertheless, a search for an ideal subtraction method that can be easily generalized to a process of arbitrary complexity continues.

- ``qt''-slicing [Catani, Grazzini]
- ``Antenna'' subtraction [Gehrmann-de Ridder, Gehrmann, Glover]
- ``Jettiness'' slicing [Boughezal et al, Gaunt et al]
- ``Sector decomposition & FKS'' subtraction [Anastasiou et al., Czakon, Heymes, Caola, Röntsch, K.M.]
- ``Projection-to-Born'' [Cacciari et al.]
- ``Colorful NNLO'' [Del Duca et al.]

THREE PILLARS OF THE SUBTRACTION SCHEMES

The so-called subtraction technique rests on three pillars:

1) Singularities in real emission contributions can only originate from soft emissions and collinear kinematic configurations;

2) Scattering amplitudes exhibit universal factorisation properties in soft and collinear limits;

$$S_4 F_{\rm LM}(1,2,4) = {\rm Eik}(1,2,4) F_{\rm LM}(1,2) \qquad C_{41} F_{\rm LM}(1,2,4) = \frac{1}{s_{14}} P\left(\frac{E_1}{E_1 - E_4}\right) F_{\rm LM}(1-4',2), \quad p_{4'} = \frac{E_4}{E_1} p_1$$

3) Perturbative computations are applied to infra-red/collinear safe observables.

$$C_{41}\mathcal{O}(1,2,4) = \mathcal{O}(1-4',2) \qquad S_4\mathcal{O}(1,2,4) = \mathcal{O}(1,2)$$

We then subtract and add back approximate cross sections in both soft and collinear limits; the difference of the full and approximate is finite and integrable and the approximate itself can be integrated over energies and angles of additional patrons since they decouples from matrix elements and observables.

$$d\sigma_R = \langle F_{\rm LM}(1,2,4)\rangle = \langle (I - S_4) F_{\rm LM}(1,2,4)\rangle + \langle S_4 F_{\rm LM}(1,2,4)\rangle$$

$$\left\langle \hat{S}_4 F_{\rm LM}(1,2,4)\right\rangle = \left\langle \int [dp_4]{\rm Eik}(1,2,4)\ F_{\rm LM}(1,2)\right\rangle = \left\langle \left(\frac{a_2}{\epsilon^2} + \frac{a_1}{\epsilon} + a_0\right) F_{\rm LM}(1,2)\right\rangle$$

REAL EMISSION CORRECTIONS: A LOOK AT NNLO

At NNLO QCD, the situation is more complex because of a non-trivial interplay between various singular limits; the basic principles remain, however, the same. For the DY process, the finite (fully-subtracted) term looks as follows and can be evaluated numerically. All subtraction terms can be integrated analytically. Comparison of numerical and known analytic results shows perfect agreement.

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}}_{1245,f_a f_b} = \sum_{(ij)\in dc} \Big\langle \left[I - \mathbb{S}\right]\left[I - S_5\right]\Big[(I - C_{5j})(I - C_{4i})\Big] \times \\
\times\, [dg_4][dg_5]\, w^{4i,5j} F_{LM,f_a f_b}(1,2,4,5)\Big\rangle \\
+ \sum_{i\in tc} \Big\langle \left[I - \mathbb{S}\right]\left[I - S_5\right]\Big[\theta^{(a)}\left[I - \mathbb{C}_i\right]\left[I - C_{5i}\right] + \theta^{(b)}\left[I - \mathbb{C}_i\right]\left[I - C_{45}\right] \\
+ \theta^{(c)}\left[I - \mathbb{C}_i\right]\left[I - C_{4i}\right] + \theta^{(d)}\left[I - \mathbb{C}_i\right]\left[I - C_{45}\right]\Big] \\
\times\, [dg_4][dg_5]\, w^{4i,5j} F_{LM,f_a f_b}(1,2,4,5)\Big\rangle .
\end{aligned}
$$

SUMMARY

Precision physics at the LHC evolves into an important tool to discover physics beyond the Standard Model. Precision leads to new opportunities, not fully appreciated earlier.

Perturbative computations move forward very rapidly; NNLO QCD computations for 2 -> 1 and 2->2 processes have been performed and can be used to do real physics.

Methods for loop computations are being developed with an eye of new challenges that we will face (multi-scale problems, masses etc). Numerical methods are getting as important as the analytic ones.

An important next challenge are the 2 -> 3 processes (3jet, ttH, tt+jet, H+2jet, Z+2jets etc.) whose understanding at NNLO QCD will require computation of missing loop amplitudes.

NNLO subtraction/slicing schemes are practically a ``done deal'' but these methods will benefit from becoming more transparent, physical and efficient.